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Investigations Concerning the Modelling of Switched Reluctance Motor Drives

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1. Introduction

At present Switched Reluctance Motors (SRM) are ones of the most preferable driving motors due to their advantages versus other electrical motors. In this frame the robustness and reliability are remarkable and make SRM drives as the most cheapest ones. The unipolar current requirements result in an equally robust power PWM converter. An important characteristic of the SRM is the higher torque/inertia ratio versus similarly sized induction or PM synchronous motors. For all that the SRM modelling is difficult due to its nonlinear magnetic circuit. This make the simulation of the motor to be possible only with major simplicity assumptions. These ones refer to the linearity of magnetic circuit, to sinusoidal variation of the electromagnetic torque with rotor angle, neglecting the leakage flux, hysteresis phenomena and iron loses etc. With these restrictions the SRM behaviour may be simulated taking into account the linearized model [1], but simulation results are in general unsatisfactory.

The paper proposes a nonlinear model of the SRM, taking into account the magnetization curves which are determined by experimental trial. With a complete set of flux-linkage values versus phase current and rotor positions, the modelling is possible by the use of a powerful environment of investigation like MATLAB [4]. In this propose an experimental trial is made on the basis of current decay for a given position of the rotor. From the magnetization curves as a function $\Psi(I,\theta)$, with usual notations, magnetic coenergy and electromagnetic torque developed by a motor phase may be calculated. Using also the voltage equation of the motor phases, a nonlinear model of the SRM is deduced. This is quite close to the reality and results in satisfactory simulation results.

2. Experimental investigation

In order to obtain magnetization curves the method of phase current decay is used and its digitally storing at a phase disconnecting (OFF state of the phase). In this purpose a set-up arrangement is considered, where the motor phase is connected to the DC source. The current decay control is given by labVIEW environment loaded on a PC[2]. Figure 1 depicts the block diagram of the electrical part of the set-up arrangement used for measuring and storing the phase current decay [3].

The main SRM characteristics are:

- no. of poles : 8/6
- no. of phases : m = 4
- rated current : I_n = 6A
- rated voltage: U_n = 60 V
- rated torque: M_n = 3Nm
- stroke angle: θ_p = 15°

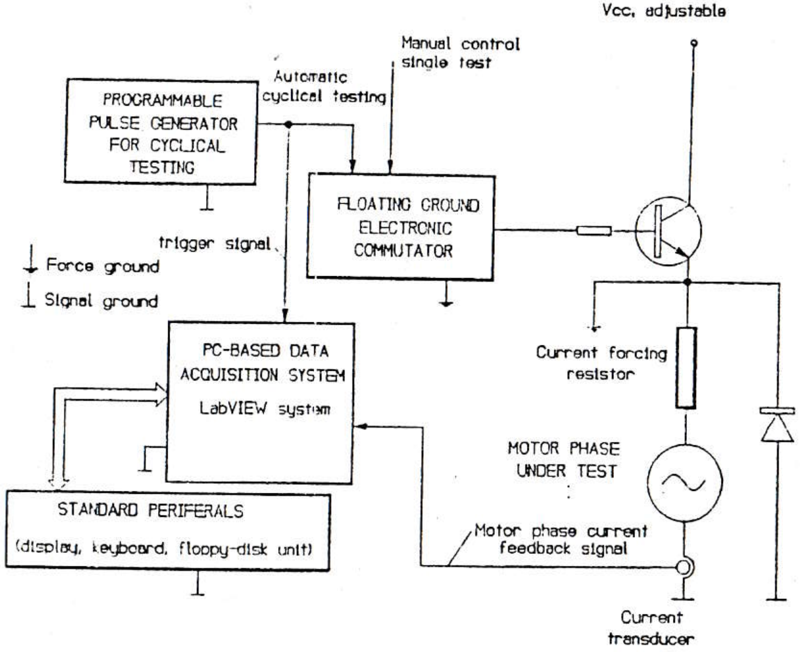

Fig. 1. Measurement of current decay.

The SRM is locked on a rotary table, which enables the measurement of rotor positions between $\theta=0°$ and $\theta=30°$ (half of a polar-step) with a step of 3°. The registration of the current decay is made for various initial phase current between I=1A and I=9A. The symmetry of motor characteristics with respect to rotor angle is taken into consideration. For instance in figure 2 is shown the waveform of the current with respect to N, as number of sampling periods equally to 0.1 ms (10 kHz). The LabWIEV environment creates an ASCII file for each current waveform (totally 9 currents x 10 positions = 90 waveforms).

Each file is taken by MATLAB environment for data processing. First an approximation of acquisitioned results is made using exponential functions provided by Optimization Toolbox. Figure 3 shows the current decay curve obtained by this procedure for a given rotor position. In this purpose it is used the Gauss-Newton least square method of approximation. This is present in Optimization Toolbox as function *leastsq* with an argument which calls an exponential sum [4].

The flux-linkage variation is obtained from the voltage equation of the phase k:

$$V_k = R_k i_k + \frac{d\psi_k}{dt} \tag{1}$$

in which usual notations appear. Along current decay period, the flux-linkage of phase k is given by the expression [5]:

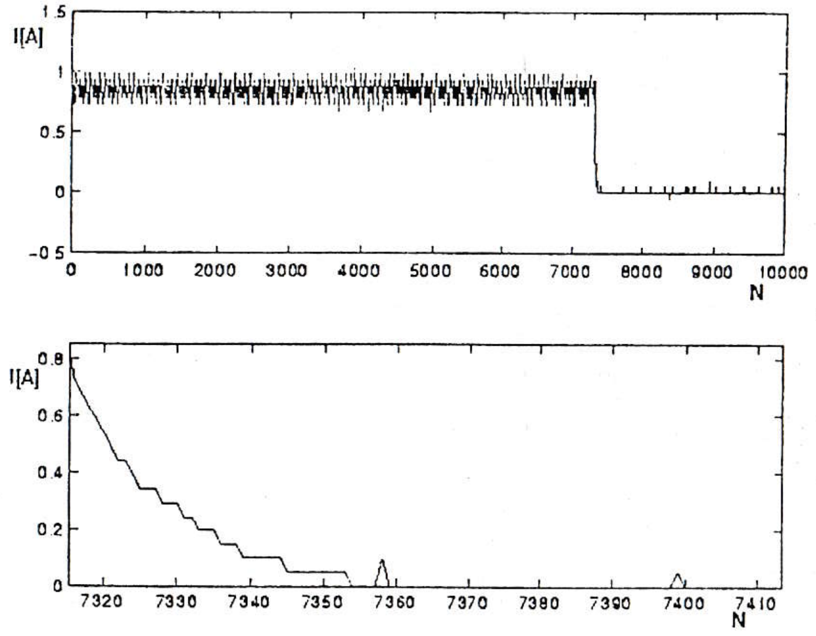

Fig. 2. Phase current decay.

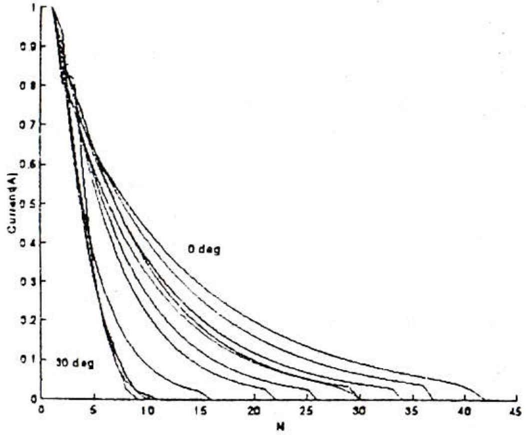

Fig. 3. Current decay curves.

$$\psi_k = \int (V_k - R_k i_k) dt \quad (2)$$

where V_k =0 and R_k is the sum of resistances along the decaying circuit.

3.Magnetization curves

The family of curves $\Psi_k(i_k,\theta)$ is obtained from bringing together the results processed by MATLAB, as shown in figure 4.

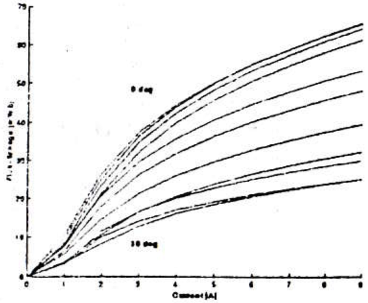

Fig. 4. Magnetization curves.

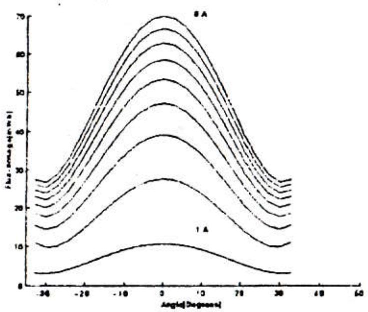

Fig. 5. Extended megnetization curves.

The variation of flux-linkage with respect to an entire pole-step of 60° is given in figure 5, which is an extension of previous given curves, on the basis of symmetry of magnetic circuit with respect to each phase. In a spatial representation the surface $\Psi_k(i_k,\theta)$ looks like in figure 6.

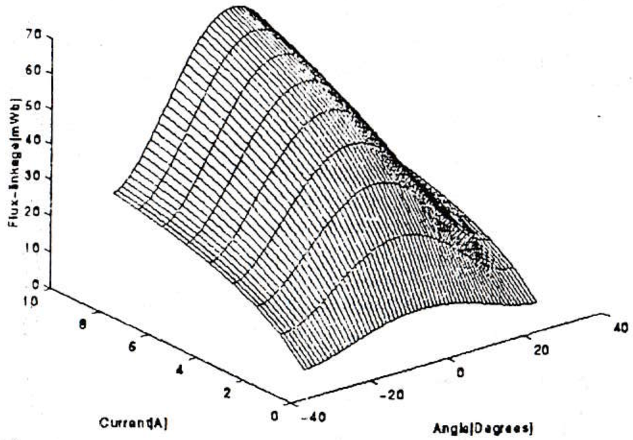

Fig. 6. Magnetization surface.

Logarithmic approximation in *leastsq* function from Optimization Toolbox is used in order to interpolate data from figure 4. In order to represent data in figure 5 it is used a polynomial approximation. The effect of saturation of magnetic circuit is observed starting with medium phase current magnitudes.

4. Magnetic coenergy

In order to deduce the contribution of each phase to developed electromagnetic torque, magnetic coenergy is calculated with the expression [6]:

$$W_k = \int_0^i \Psi_k(i_k\theta_k)di_k \qquad (3)$$

which represents the aria below $\Psi_k(i_k)$ curves until a given current for each rotor position. Adaptive recursive Simpson's rule is used for calculating the integral (3).

Figure 7 shows the variation of coenergy with respect to rotor angle for different phase currents. The coenergy increases with phase current for a given angular position, but the effect of saturation is also observed.

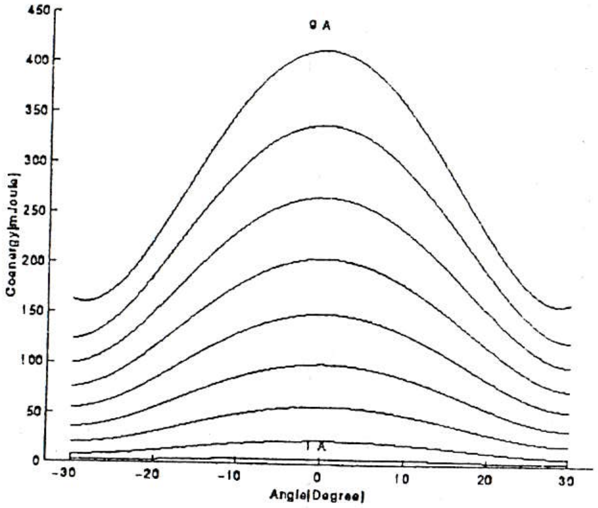

Fig. 7. Coenergy curves.

5. Electromagnetic torque

The expression of electromagnetic torque results by differentiating the expression [3]:

$$M_k(i_k,\theta) = \frac{dW_k}{dt}\Big|_{i_k = const.} \qquad (4)$$

which represents the theorem of generalized forces applied in case of SRM. A polynomial differential was used in order to obtain torque versus angle variation.

Figure 8 depicts the electromagnetic torque variation as function of phase current and rotor angle. As expected, the variation of torque with rotor angle is not sinusoidal, but symmetrical with respect to aligned rotor positions.

The electromagnetic torque developed by SRM is deduced as a sum of contributions of each phase, as follows:

$$M = \sum_{k=1}^{4} M_k(i_k,\theta) \qquad (5)$$

This function is useful for SRM simulation if it is associated to rotor torque equilibrium and also to dynamic equation of voltages across each phase. The total torque developed by SRM is depicted in figure 9. Relatively small torque ripples are observed.

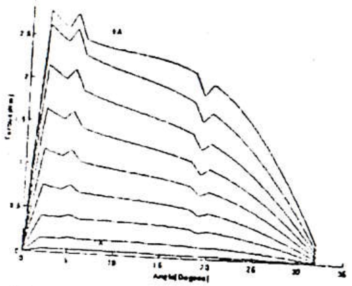

Fig. 8. Electromagnetic torque per phase curves.

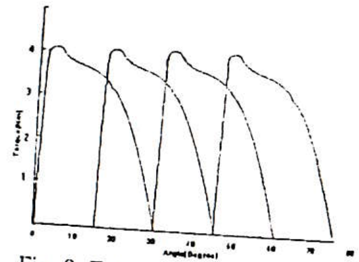

Fig. 9. Total electromagnetic torque.

5.Conclusions

Mathematical modelling of SRM can be successfully accomplished using as start real magnetization curves from experimental investigations. As a result numerical simulation based on nonlinear parameters is more adequate than classical approach, due to its simplicity and also to its real basis. Dynamic equations of the SRM are of simpler structure that avoid to explicit flux-linkage as function of inductivities. Also it includes the major nonlinearity due to magnetic saturation. Moreover, the mathematical model as described above offers a good interface to MATLAB environment for an elegant simulation of SRM behaviour.

References

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