Sinusoidal Oscillators

- Signal generators: sinusoidal, rectangular, triangular, TLV, etc.

- Obtaining a sine wave:
  - triangle \( \xrightarrow{\text{functional transf.}} \) sine
  - sine wave generation: frequency selective network in a feedback loop of a PF amplifier: *sinusoidal oscillator*

- Oscillation frequency: \( f_0 \)
- Oscillation amplitude: \( \hat{V}_o \)
- Oscillation criterion
- Frequency stability
- Amplitude stability
- Distortion coefficient
Oscillator feedback loop

PF amplifier:

\[ x_i = x_s + x_r \]

\[ A = \frac{a}{1 - ar} \]

\[ x_s = 0; \quad x_0 = \text{finit} \]

\[ A(j\omega) = \frac{a(j\omega)}{1 - a(j\omega)r(j\omega)} \]

\[ 1 - a(j\omega_0)r(j\omega_0) = 0 \]
Oscillation criterion

Barkhausen’s criterion

\[ a(j\omega_0)r(j\omega_0) = 1 \]

Signal reconstruction on the feedback loop

\[ a(j\omega) = |a(j\omega)|e^{j\varphi_a} \]
\[ r(j\omega) = |r(j\omega)|e^{j\varphi_r} \]

\[ a(j\omega_0)r(j\omega_0) = |a(j\omega_0)|r(j\omega_0)|e^{j(\varphi_a + \varphi_r)} = 1 \]

module condition: \[ |a(j\omega_0)|r(j\omega_0)| = 1 \] gives \( a_0 \)

phase condition: \[ \varphi_a + \varphi_r = 2k\pi \] gives \( f_0 \)

Who sets \( \hat{V}_o \)? Nonlinearity of the gain
\[ |a(j\omega)| |r(j\omega)| < 1 \quad \text{oscillations are attenuated - zero} \]
\[ |a(j\omega)| |r(j\omega)| > 1 \quad \text{oscillations are amplified - saturation} \]

Stability of the oscillation amplitude

Automatic gain control

\[ v_o = \hat{V}_o \sin 2\pi f_0 t \]
\[ |r(j\omega)| = \text{cst} \]
\[ \hat{V}_o \uparrow, |a(j\omega_0)| \downarrow, \hat{V}_0 \downarrow \]
RC Oscillators

- Basic amplifier independent of frequency:
  - inverting $\varphi_r = -180^\circ$
  - noninverting $\varphi_r = 0$

- Frequency selective network
  BPF with one zero and two poles

  Phase shift of the network lies in the range of $[+90^\circ; -90^\circ]$.

  We want $\varphi_r = 0$ just for a single frequency $f_0$

  It is necessary to bring closer the two poles.
For only one single frequency, \( f_0 \) we have

\[ \varphi_r = 0 \]

\[
F(j\omega) = \frac{v_r(j\omega)}{v_o(j\omega)}
\]
WIEN Bridge—cont.

\[
\begin{align*}
    r(j\omega) &= \frac{v_r(j\omega)}{v_o(j\omega)} = \frac{1}{1 + \frac{R_s}{R_p} + \frac{C_p}{C_s} + j\left(\omega R_s C_s - \frac{1}{\omega R_p C_p}\right)} \\
    \varphi_r &= 0 \\
    \omega_0 R_s C_s - \frac{1}{\omega_0 R_p C_p} &= 0
\end{align*}
\]

\[
\begin{align*}
    f_0 &= \frac{1}{2\pi \sqrt{R_s R_p C_s C_p}} \\
    R_s &= R_p = R \\
    C_s &= C_p = C
\end{align*}
\]

\[
\begin{align*}
    f_0 &= \frac{1}{2\pi RC} \\
    |r(j\omega_0)| &= \frac{1}{1 + \frac{R_s}{R_p} + \frac{C_p}{C_s}} \\
    |r(j\omega_0)| &= \frac{1}{3}
\end{align*}
\]
Op amp and WIEN bridge oscillator

\[ v_o(t) = \hat{V}_o \sin 2\pi f_0 t \]

\[ R_s = R'_p = R \]

\[ C_s = C'_p = C \]

\[ f_0 = \frac{1}{2\pi RC} \]

\[ |r(j\omega_0)| = \frac{1}{3} \]

\[ |a(j\omega_0)| = \frac{1}{|r(j\omega_0)|} = 3 \]

\[ a = 1 + \frac{R_4}{R_3} \]

\[ 1 + \frac{R_4}{R_3} = 3 \]

\[ R_4 = 2R_3 \]

\[ \hat{V}_o = ? \]

Nonlinearity on the gain, close to saturation
Automatic control of the amplitude

1. Using diodes

\[ a = 1 + \frac{R_4' + R_4''}{R_3} \| r_d \]

for \( v_o(t) \) small, \( D_1, D_2 - (off) \)

\[ a = 1 + \frac{R_4' + R_4''}{R_3} \]

\[ |a(j\omega_0)||r(j\omega_0)| > 1 \]

\( v_o(t) \) increases, \( D_1 - (on) \) on the positive half-cycle, \( D_2 - (on) \) on the negative half-cycle

\[ \hat{V}_o \] is given by the value of \( r_d \) to maintain oscillations

\[ \hat{V}_o \] is given by the value of \( r_d \) to maintain oscillations
2. Using n-channel depletion-type MOSFET

\[ a = 1 + \frac{R_4}{R_3 + r_{DS}} \]

\[ r_{DS} \approx \frac{1}{2\beta(v_{GS} - V_{Th})} \]
Op amp and RC ladder network oscillator

- High pass band
- Low pass band

The phase-shift is in the range of \([0^\circ; -90^\circ]\)
- Inverting basic amplifier
- How many identical RC cells are necessary to build an oscillator?
low pass RC ladder with 3 cells

\[ r(j\omega) = \frac{1}{1 - 5(\omega RC)^2 + j[6\omega RC - (\omega RC)^3]} \]

\[ \varphi_r = 0 \]

\[ 6\omega_0 RC - (\omega_0 RC)^3 = 0 \]

\[ f_0 = \frac{\sqrt{6}}{2\pi RC} \]

\[ r(j\omega_0) = -\frac{1}{29} \]
The circuit of RC ladder network oscillator

Why the basic amplifier does not contain only one inverting op-amp amplifier?
How does the voltages $v_o(t)$ and $v_+(t)$ look like in the steady-state regime? What is the oscillation frequency?

Size $R_4$ so that the circuit will maintain the oscillation. In conduction assume the equivalent diode resistance $r_{D1}=r_{D2}=0.5\,\text{K}\Omega$. Verify if the oscillation can start.

How does the voltage $v_o(t)$ look like in the steady-state regime if $D_2$ diode is missing in the circuit?