Nonsinusoidal Signal Generators

- rectangle, triangle, saw tooth, pulse, etc.

Multivibrator circuits:

- **astable** – no stable states (two quasi-stable states; it remains in each state for predetermined times)

- **monostable** – one stable state, one non-stable state

- **bistable** – two stable states

  - From one stable state the circuit switches in the other state under the action of a control signal (input signal).
  - From one non-stable state the circuit switches in the other state automatically.
$C$ in the time domain

Defining relation between current and voltage

$$C \frac{dv_C(t)}{dt} = i_C(t)dt$$
RC circuit – time domain analysis

- RC circuit with voltage source

\[ Ri_C(t) + v_C(t) = v_I(t) \]

\[ C \frac{dv_C(t)}{dt} = i_C(t) dt \]

\[ i_C(t) = C \frac{dv_C(t)}{dt} \]

\[ RC \frac{dv_C(t)}{dt} + v_C(t) = v_I(t) \]

\[ \tau = RC \quad \text{time constant of the circuit} \]

\[ v_C(t) = v_C(0)e^{-\frac{t}{\tau}} + (1 - e^{-\frac{t}{\tau}})v_C(\infty) \]
RC circuit with dc voltage source

\[ v_c(t) = v_c(0)e^{-\frac{t}{\tau}} + (1 - e^{-\frac{t}{\tau}})v_c(\infty) \]

\[ v_c(0) = 0; \quad v_c(\infty) = V_I; \quad \tau = RC \]
Charging the $C$ with a constant current

$$Cd v_c(t) = i_c(t) dt$$

$$v_c(t) = \frac{1}{C} \int_0^t i_c(t) dt + v_c(0)$$

$v_c(t) = \frac{1}{C} It + v_c(0)$

Diagram:
- Current $i_c$ flowing into the capacitor
- Voltage $v_c$ increasing linearly with time $t$
- Slope $\frac{1}{C} I$
Charging the capacitor - summary

- with a *R* and a DC voltage source

\[ v_c(t) = v_c(0)e^{\frac{-t}{\tau}} + (1 - e^{\frac{-t}{\tau}})v_c(\infty) \]

- with a constant current source

\[ v_c(t) = \frac{1}{C}It + v_c(0) \]

The transient regime occurs from 0 to \( \tau \) and stabilizes in the steady-state regime.

Transient regime

Steady-state regime

\[ 0.63V_I \]

\[ V_I \]
Astable multivibrators (Relaxation oscillators)

Operating principle

- the time variation of the voltage across the capacitor is exponential type

- if the voltage across the capacitor is fed to a PF comparator, a rectangular wave is obtained
1. Astable multivibrator - rectangular signal generator

\[ v_C(t) = v_C(0) e^{-\frac{t}{\tau}} + \left(1 - e^{-\frac{t}{\tau}}\right) v_C(\infty) \]

\[ v_D(t) = v^+ - v^- = \frac{R_1}{R_1 + R_2} v_O(t) - v_C(t) \]

\[ V_{THH} = \frac{R_1}{R_1 + R_2} V_{OL} = r V_{OL} \]

\[ V_{THL} = \frac{R_1}{R_1 + R_2} V_{OH} = r V_{OH} \]
\[
t \in (t_1, t_2) \quad V_{ThH} = V_{ThL}e^{\frac{-T_c}{\tau}} + \left(1 - e^{\frac{-T_c}{\tau}}\right)V_{OH}; \quad T_c = \tau \ln \frac{V_{OH} - rV_{OL}}{(1 - r)V_{OH}}
\]

\[
t \in (t_2, t_3) \quad V_{ThL} = V_{ThH}e^{\frac{-T_d}{\tau}} + \left(1 - e^{\frac{-T_d}{\tau}}\right)V_{OL}; \quad T_d = \tau \ln \frac{rV_{OH} - V_{OL}}{(r - 1)V_{OL}}
\]

Generally \( V_{OH} = -V_{OL} \)

\[
T_c = T_d = \frac{T}{2} = \tau \ln \frac{1 + r}{1 - r}
\]

\[
T = 2RC \ln \frac{1 + r}{1 - r}
\]

If \( R_1 = R_2 \)

\[
T = 2RC \ln 3 \approx 2,2RC
\]
Problem

\[ \pm V_{PS} = \pm 12\text{V}, \ R_1=10\text{k}\Omega, \ R_2=20\text{k}\Omega, \ R=7.5\text{k}\Omega \text{ and } C=10\text{nF}. \text{ The op amp is a rail-to-rail type.} \]

a) What are the minimum and maximum values for the voltage across the capacitor?
b) What is the frequency of the rectangular signal?
c) Modify the circuit for an adjustable frequency between \( f_{\text{min}}=0.8\text{kHz} \) and \( f_{\text{max}}=8\text{kHz} \)?

\[
V_{ThL} = \frac{R_1}{R_1 + R_2} V_{OL} = \frac{10}{10 + 20}(-12) = -4\text{V}
\]

\[
V_{ThH} = \frac{R_1}{R_1 + R_2} V_{OH} = \frac{10}{10 + 20} \cdot 12 = 4\text{V}
\]
b) \[ r = \frac{R_1}{R_1 + R_2} = \frac{10}{10 + 20} = \frac{1}{3} \]

\[ T = 2RC \ln \frac{1 + r}{1 - r} = 2 \cdot 7.5k\Omega \cdot 10nF \cdot \ln \frac{1 + 1/3}{1 - 1/3} = 103.5\mu s \]

\[ f = \frac{1}{T} = \frac{1}{166} = 9.7kHz \]

c) \[ T = 2RC \ln \frac{1 + r}{1 - r} = 2RC \ln 2 = 1.386 RC \]

\[ T_{\text{min}} = \frac{1}{f_{\text{max}}} = 1.386R'C \]

\[ R' = \frac{1}{1.386f_{\text{max}}C} = \frac{1}{1.386 \cdot 8kHz \cdot 10nF} = 9k\Omega \quad R' = 8.87k\Omega (1%) \]

\[ T_{\text{max}} = \frac{1}{f_{\text{min}}} = 1.386(R' + P)C \quad P = 9R' = 9 \cdot 8.87 = 79.8k\Omega \]

\[ P = 100k\Omega \]
2. Astable multivibrator with an integrator and a comparator

Rectangular and triangular signal generator

\[ v_{o1}(t) = v_c(t) \]

\[ V_{ThL} = -\frac{R_1}{R_2} V_{OH} \]

\[ V_{ThH} = -\frac{R_1}{R_2} V_{OL} \]
\[ C \Delta v_C = i_C \Delta t \]
discharge
\[ i_C = \frac{0 - V_{OH}}{R} = -\frac{V_{OH}}{R} \]
\[ \Delta v_C = V_{ThL} - V_{ThH} ; \]
\[ \Delta t = t_2 - t_1 = T_d \]
\[ T_d = RC \frac{V_{ThH} - V_{ThL}}{V_{OH}} \]
\[ T_c = RC \frac{V_{ThH} - V_{ThL}}{-V_{OL}} \]

In general \[ V_{OH} = -V_{OL} \]

\[ T = 2RC \frac{V_{ThH} - V_{ThL}}{V_{OH}} = 4RC \frac{R_1}{R_2} \]

If \[ R_1 = R_2 \]

\[ T = 4RC \]
\[ f = \frac{1}{4RC} \]
At saturation the output voltage of op-amps is within 1V of the supply

a) What is the amplitude of the triangular voltage?
b) What is the oscillation frequency?
c) What is the maximum value of the current to the output of each op amp?
The independence of the supply voltage

\[ V_{OH} = V_Z + 0.7V; \quad V_{OL} = -V_Z - 0.7V \]

The reverse-biased base to emitter junction behaves as a Zener diode, regulating the voltage at a voltage dependent on the transistor type and on the emitter current (5V … 8V).
Frequency adjustment

\[ T = 2RC \frac{V_{ThH} - V_{ThL}}{V_{OH}} \]

\[ V_{ThL} = - \frac{R_1}{R_2} V_{OH}; \quad V_{ThH} = - \frac{R_1}{R_2} V_{OL}; \quad V_{OH} = -V_{OL}; \quad V_{ThH} - V_{ThL} = 2 \frac{R_1}{R_2} V_{OH} \]

\[ T = 2RC \frac{R_1}{R_2} \frac{V_{OH}}{V_{OH}} = 4RC \frac{R_1}{R_2} \frac{V_{OH}}{V_{OH}} \]

\[ f = \frac{1}{4RC} \frac{R_2}{R_1} \frac{V'_{OH}}{V_{OH}} \]

If \( V'_{OH} \) can be adjusted, than the period (frequency) can be adjusted.
**Frequency adjustment - cont.**

If \( v'_o = v_o \);

\[
f_{\text{max}} = \frac{1}{4RC} \frac{R_2}{R_1} ;
\]

If \( v'_o = \frac{R_4}{R_4 + P_f} v_o \);

\[
f_{\text{min}} = \frac{1}{4RC} \frac{R_2}{R_1} \frac{R_4}{R_4 + P_f} .
\]
Problem

Consider rail-to-rail op-amps
a) What is the oscillation frequency?
b) Modify the circuit to obtain an adjustable frequency in the range of [5; 25] kHz.
Offset adjustment of the triangular voltage

From Pa one can move the hysteresis along the horizontal axis, thus adjusting the offset (dc level) of the triangular voltage.

\[ V_{ThL} = \left( 1 + \frac{R_1}{R_2} \right) V_a - \frac{R_1}{R_2} V_{OH} \quad \quad V_{ThH} = \left( 1 + \frac{R_1}{R_2} \right) V_a - \frac{R_1}{R_2} V_{OL} \]

\[ V_{a_{max}} = \frac{P_a + R_6}{R_5 + P_a + R_6} V_{PS} + \frac{R_5}{R_5 + P_a + R_6} (-V_{PS}) = 3.75V \]
Offset adjustment of the triangular voltage

\[ V_{a_{\text{min}}} = \frac{R_6}{R_5 + P_a + R_6} V_{PS} + \frac{P + R_5}{R_5 + P_a + R_6} (-V_{PS}) = -3.75 \text{V} \]

The offset can be adjusted between

\[ V_{o1_{\text{max}}} = \left(1 + \frac{11}{33}\right) \cdot 3.75 = 5 \text{V}; \quad V_{o1_{\text{min}}} = \left(1 + \frac{11}{33}\right) \cdot (-3.75) = -5 \text{V} \]
Specialized integrated circuits for signals generation

- **8038** - Precision Waveform Generator/Voltage Controlled Oscillator (triangular, square, sine, sawtooth, pulse) **OBSOLETE PRODUCT**

- **NE566** - Function generator VCO, square, triangular - 1MHz

- **AD9833** - Low power, programmable waveform generator: sine, triangular, and square wave. No external components. Frequency and phase are software programmable. 3-wire serial interface. Power-down function (SLEEP). 0 MHz to 12.5 MHz output frequency range.

- **555** - highly stable device for generating accurate time delays or oscillation (astable and monostable)
Clock generators

- Quartz-crystal oscillator
  
  \[ f_0 = 1, 2, 4, 5, \ldots, 20\text{MHz} \]

  \[ f_0 = 14,31818\text{MHz} \] - video adapter in personal computers

  \[ f_0 = 32,768\text{KHz} \] - digital wrist watch, divide by \(2^{15}\) to get 1Hz

- NOT gates oscillator

\[ \text{Optional} \]