Frequency Response of Transistor Amplifiers

- Frequency response of CS amplifier, qualitative analyses
- Quantitative analyses of frequency response for CS and CE amplifiers - optional
- Cascode amplifier - optional
CS amplifier

Medium frequency

Medium frequency small-signal equivalent circuit

- $A_v \approx -g_m R_D$
- $R_i = R_G$
- $R_O = R_D \parallel r_o \approx R_D$
Frequency response of transistor amplifiers

➢ **mid-frequency:**
  - coupling capacitors → short-circuits
  - internal parasitic capacitances → open-circuits

➢ **low-frequency:**
  - coupling capacitors → equivalent impedances
  - internal parasitic capacitances → open-circuits

➢ **high-frequency:**
  - coupling capacitors → short-circuits
  - internal parasitic capacitances → equivalent impedances

• **must be taken into consideration:**
  - output resistance of the signal source
  - load resistance
**Mid-frequency**

- No capacitors in the small-signal equivalent circuit

- **Frequency independent behavior**

\[ |A_v(j\omega)| = \text{cst} \]

against frequency variations
Low-frequency

- Coupling capacitors in the small-signal equivalent circuit
- Frequency dependent behavior
- High pass type

\[ f \downarrow, \quad |A_v(j\omega)| \downarrow \]

Sets the lower cutoff frequency \( f_L \)
High-frequency

- Parasitic capacitors in the small-signal equivalent circuit
- Frequency dependent behavior
- Low pass type

\[ f \uparrow, \quad \left| A_v(j\omega) \right| \downarrow \]

Sets the upper cutoff frequency \( f_H \)
Frequency response

Due to coupling capacitances

Due to parasitic capacitances

Band pass
CS amplifier

Load

Analysis in the low-frequency range

\[
A_v(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)}
\]

\[
v_o(j\omega) = F_o(j\omega) \cdot i_d(j\omega);
\]

\[
i_d(j\omega) = F_s(j\omega) \cdot v_g(j\omega);
\]

\[
v_g(j\omega) = F_i(j\omega) \cdot v_i(j\omega);
\]

\[
A_v = F_i(j\omega) \cdot F_s(j\omega) \cdot F_o(j\omega)
\]
Analysis in low-frequency range

Optional

$$F_i(j\omega) = \frac{v_g(j\omega)}{v_i(j\omega)}$$

$$F_i(j\omega) = \frac{j\omega R G C_{ci}}{1 + j\omega (R + R_g) C_{ci}}$$

HPF, introduces a pole at the frequency

Frequency of the pole: $C_{ci}$ multiplied by the total resistance seen by $C_{ci}$

Generalization:

$$f_{Li} = \frac{1}{2\pi(R + R_g)C_{ci}}$$
Analysis in low-frequency range

\[ F_s(j\omega) = \frac{i_d(j\omega)}{v_g(j\omega)} \]

\[ i_d(j\omega) = g_m v_{gs}(j\omega) \]

\[ v_g(j\omega) = v_{gs}(j\omega) + g_m v_{gs}(j\omega) \left( R_s \parallel \frac{1}{j\omega C_s} \right) \]

Frequency of the pole: \( C_s \) multiplied by the total resistance seen by \( C_s \)

Generalization:

\[ f_{ls} = \frac{1}{2\pi \left( R' \parallel R_s \right) C_s} \]
Analysis in low-frequency range

\[ F_o(j \omega) = \frac{v_o(j \omega)}{i_d(j \omega)} \]

Frequency of the pole: \( C_{Co} \) multiplied by the total resistance seen by \( C_{Co} \)

Generalization:

\[ f_{Lo} = \frac{1}{2\pi(R_D + R_L)C_{Co}} \]

- **Dominant pole**: the greater break frequency between \( f_{Li}, f_{Ls}, f_{Lo} \) if the nearest pole or zero will be at least a decade away.
- Usually given by \( f_{Ls} \), for equal coupling capacitances
➢ Analysis in the high-frequency range

$C_{gd}$ is reflected to the input according to Miller’s theorem

\[ C_{gd,ech} = (1 - a_v)C_{gd} \]
\[ a_v = -g_m R'_L = -g_m \left( R_L \parallel R_D \parallel r_o \right) \]

Optional

$C_{ds}$ is not shown here because it generates a pole at a much higher frequency than the one generated by $C_{gs}$ and $C_{gd}$
Analysis in the high-frequency range

\[ C_i = C_{gs} + C_{gd,ech} = C_{gs} + (1 + g_m R'_L) C_{gd} \]

\[ A_v(j \omega) = \frac{v_o(j \omega)}{v_i(j \omega)} = - \frac{R_G}{R + R_G} g_m R'_L \frac{1}{1 + j \omega (R || R_G) C_i} \]

\[ A_{vo} = - \frac{R_G}{R + R_G} g_m R'_L \]

\[ f_H = \frac{1}{2 \pi (R || R_G) C_i} \]

\[ f_H = \frac{1}{2 \pi (R || R_i) C_i} \]
Numerical example

\[ C_{Ci} = C_{Co} = C_s = 10 \mu F, \ R = 20K\Omega, \ R_G = 2M\Omega, \ R_D = 10k\Omega, \ R_L = 20k\Omega, \ R_s = 10K\Omega, \ I = 400\mu A. \]

\[ K = 100\mu A/V^2, \ (W/L) = 18, \ V_A = 100V. \quad C_{gs} = C_{gd} = 1pF \ @ \ I = 400\mu A \]

**Solution:**

\[ g_m = 1.2\text{mS} \quad r_o = 250K\Omega \]

\[ f_{Li} = \frac{1}{2\pi(R + R_G)C_{Ci}} = \frac{1}{2\pi(20 + 2000)10^3 \cdot 10 \cdot 10^{-6}} \approx 8\text{mHz} \]

\[ f_{Ls} = \frac{1}{2\pi \left( \frac{1}{g_m} \parallel R_S \right) C_s} = \frac{1}{2\pi \left( \frac{1}{1,2} \parallel 10 \right) 10^3 \cdot 10 \cdot 10^{-6}} \approx 21\text{Hz} \]

\[ f_{Lo} = \frac{1}{2\pi(R_D + R_D)} = \frac{1}{2\pi(10 + 20)10^3 \cdot 10 \cdot 10^{-6}} \approx 0.5\text{Hz} \]

\[ f_L = 21\text{Hz} \]

The output resistance of the signal source \( R \) does not affect \( f_{Li} \) but affects \( f_H \).
\[ A_{vo} = -\frac{R_G}{R + R_G} g_m R_L = -\frac{2}{0.02 + 2} \cdot 1.2 \cdot 6.5 = -7.7 \]

\[ |A_{vo}|_{dB} = 20 \log(7.7) = 17.7 \]

\[ C_i = C_{gs} + [1 + g_m (r_o \parallel R_D \parallel R_L)] C_{gd} = 1 + [1 + 1.2(250 \parallel 10 \parallel 20)] \cdot 1 \approx 9.8 \text{pF} \]

\[ f_H = \frac{1}{2\pi (R \parallel R_G) C_i} = \frac{1}{2\pi (20 \parallel 2000) \cdot 10^3 \cdot 9.8 \cdot 10^{-12}} = 820 \text{KHz} \]
CE amplifier

➢ Analysis in the low-frequency range

The effect of each capacitor is analyzed considering the other two capacitors with infinite capacity (zero equivalent impedance)

\[
f_{Li} = \frac{1}{2\pi(R + R_i)C_{Ci}} = \frac{1}{2\pi(R + R_B \parallel r_{be})C_{Ci}}
\]

\[
f_{Le} = \frac{1}{2\pi \left( R' \parallel R_E \right) C_E} = \frac{1}{2\pi R_E' C_E};
\]

\[
f_{Lo} = \frac{1}{2\pi(R_O + R_L)C_{Co}} = \frac{1}{2\pi(R_C + R_L)C_{Co}}
\]

\[
R_B = R_{B1} \parallel R_{B2}
\]

\[
R'_E = R_E \parallel R' = R_E \parallel \frac{r_{be} + R_B}{\beta + 1} \parallel R
\]
Analysis in the high-frequency range

\[ A_v(j\omega) = \frac{R_B \parallel r_{be}}{R + R_B \parallel r_{be}} g_m(R_C \parallel R_L) \frac{1}{1 + j\omega R' C_i} \]

\[ C_i = C_{be} + (1 + g_m R'_L) C_{bc} \]

\[ R'_i = r_{be} \parallel R_B \parallel R \]

\[ A_{vo} = \frac{R_B \parallel r_{be}}{R + R_B \parallel r_{be}} g_m(R_C \parallel R_L) \]

\[ f_H = \frac{1}{2\pi(r_{be} \parallel R_B \parallel R) C_i} \]
Cascode amplifiers

➢ for the CS and CE amplifiers the magnitude of the gain and the bandwidth are in inverse ratio due to Miller’s effect

➢ when the gain increases, the parasitic capacitance reflected to the input also increases, so the high breaking frequency decreases

\[
A_{vo} = -\frac{R_G}{R + R_G} g_m R'_L
\]

\[
f_H = \frac{1}{2\pi \left( R \parallel R_i \right) \left( C_{gs} + (1 + g_m R'_L) C_{gd} \right)}
\]

➢ to reduce the multiplication effect of the parasitic capacitance it is often use the cascode configuration:

- connection of a CS (CE) amplifier followed by a CG (CB) amplifier

➢ a technique to build wideband amplifiers
The cascode configuration with MOSFET

\[ A_{v1} = \frac{v_{o1}}{v_i} = -g_{m1} \left( r_{o1} \parallel \frac{1}{g_{m2}} \right) \]

\[ \frac{1}{g_{m2}} \ll r_{o1} \quad \text{same current through } T_1, T_2 \]

\[ g_m = g_{m1} = g_{m2} \quad A_{v1} \approx -1 \]

\[ A_v = A_{v1} \cdot A_{v2} \]

\[ A_v = -g_m R_D \]

\[ R_i = R_{G1} \parallel R_{G2} = R_G \]

\[ R_o = R_D \parallel (r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}) \approx R_D \]
High frequency

The multiplication factor of $C_{gd1}$ is 2, considerably smaller than the one in CS configuration $(1 + g_m R_2')$. The value for $C_i$ results much more reduced, that leads to a much higher value of the high cutoff frequency:

$$f_H = \frac{1}{2\pi (R \parallel R_G) C_i}$$

- the other two poles due to $C_{gs2}$ and $C_{gd2}$ results to considerable higher frequency than $f_H$
- $C_i$ introduces the dominant pole at high frequency and determines the pass-band of the amplifier
Numerical illustration

\[ R_{G1} = R_{G2} = 4 \Omega \quad R_D = 10 \Omega \]
\[ R = 20 \Omega \quad R_L = 20 \Omega \]
\[ I = 400 \mu A \]
\[ (W / L) = 18 \quad V_A = 100V \]
\[ K = 100 \frac{\mu A}{V^2} \]
\[ C_{gs} = C_{gd} = 1 \text{pF} \]

\[ g_m = \sqrt{2K} \sqrt{\frac{W}{L}} \sqrt{I} = \sqrt{2 \cdot 100 \cdot \sqrt{18} \cdot \sqrt{400}} = 1200 \ \mu S = 1.2 \ \text{mS} \]

\[ R_G = R_{G1} \parallel R_{G2} = 4 \parallel 4 = 2 \Omega \]

\[ A_v = \frac{R}{R + R_G} \left[ -g_m(R_D \parallel R_L) \right] = -\frac{20}{20 + 2000} \cdot 1.2 \cdot (10 \parallel 20) = -7.9 \]

\[ C_i = C_{gs} + 2C_{gd} = 1 + 2 \cdot 1 = 3 \text{pF} \]

\[ f_H = \frac{1}{2\pi(R \parallel R_G)C_i} = \frac{1}{2\pi(20 \parallel 2000) \cdot 10^3 \cdot 3 \cdot 10^{-12}} \approx 2.7 \text{MHz} \]
The cascode configuration with BJT

For identical $T_1$ and $T_2$ transistors

$$R_B = R_{B1} \parallel R_{B2}$$
$$r_{be} = r_{be1} = r_{be2}$$
$$g_m = g_{m1} = g_{m2}$$

$$A_v = \frac{v_o}{v_i} = \frac{R_B \parallel r_{be1}}{R + R_B \parallel r_{be1}} \left(-g_{m1} \frac{1}{g_{m2}}\right) g_{m2} (R_C \parallel R_L)$$

$$A_v = -\frac{R_B \parallel r_{be}}{R + R_B \parallel r_{be}} g_m (R_C \parallel R_L)$$

$$f_H = \frac{1}{2\pi (r_{be} \parallel R_B \parallel R)(C_{be} + 2C_{bc})}$$