Chapter 1

FUNDAMENTALS

Introduction

This chapter analyses the fundamental notions that everyone needs when exploring the field of electronics.

We will start the chapter with the beginning, talking about electric signals and going further with the relations and theorems used in electronic circuits analysis. The voltage and current sources as well as the passive components (such as resistors, capacitors, and inductors) also find their place among the fundamental notions. Besides the time domain, the frequency domain with its representation of the frequency response of the circuits containing reactive elements is also approached.

Thus, a great part of this chapter rather assumes a review of some notions and pieces of knowledge that have been previously studied. Besides these, a part of the terminology, conventions and notations used throughout the book are presented.

At the end of this chapter we should be armed with means and working instruments just suited for understanding the operating principles of the electronic devices and their main applications.

1.1 Electric Signals

By *signal* we understand any quantity that can offer information such as: sound, image, temperature, force, speed and so on. In the electronics, we are primarily interested in *electric signals*:

- *Electric voltage,* denoted by v or V; unit of measure: volt V with its submultiples mV ($1mV=10^{-3}V$), μV ($1\mu V=10^{-6}V$);
- *Electric current,* denoted by i or I; unit of measure ampere: A with its submultiples mA ($1mA=10^{-3}A$), μA ($1\mu A=10^{-6}A$).

Depending on the particular way they change in time, electric signals are classified in:
-*continuous signals* which do not change their value in time and which are referred to as dc (direct current);
-*time-varying signals* whose value changes in time and which are referred to as ac (alternating current).

As an example, Fig. 1.1.1a) shows a continuous voltage V=5V, whereas Fig. 1.1.1b) shows a variable voltage, particularly a sine wave $v=3\sin\omega t$ V.

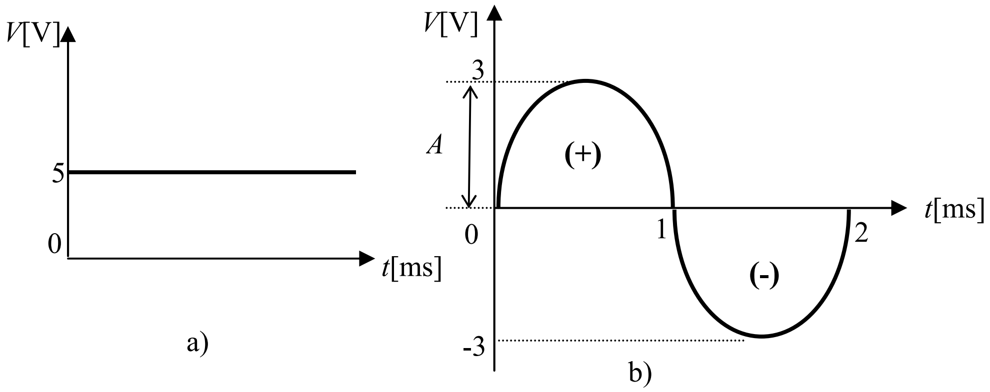


Fig. 1.1.1 The waveform of a: a) continuous voltage; b) sinusoidal voltage.

The parameters of a sinusoidal signal are:

- Amplitude: A=3V;
- Peak to peak value, which is the difference between the maximum and the minimum value of the signal: 6V;
- Root-mean-square value of the signal

$$V_{rms} = \frac{A}{\sqrt{2}} = 2.12\text{ V};$$

- Period of the signal T=2ms;
- Average value or the dc component on a time interval. For periodic signal, we calculate the average value on a period. The positive half-cycle (+) and the negative half-cycle (-) of the sinusoidal waveform being equal, the average value of the voltage is zero;
- Instantaneous value, which is the value of the signal at a certain moment in time. For example at $t=T/4$ the instantaneous value is +3V.

The sinusoidal waveforms are the most frequently used signals. Most of the electric appliances (computers, TV sets, refrigerators, etc.) are powered from the ac power line with a sinusoidal voltage with 230V value, 325V amplitude, and 50Hz frequency. In the United States of America, the ac power line has 117V root-mean-square value at a 60Hz frequency.

Remarks: When measuring a sinusoidal voltage with a voltmeter, it will indicate the root-mean-square value of the voltage.

Besides the sinusoidal wave, there are lots of other variable signals, the most common being: triangle wave, square wave, sawtooth wave, positive or negative pulse, step, and spike signals.

- **Signal Sources. Notations**

For electric signal sources, we will use the symbols shown in Fig. 1.1.2.

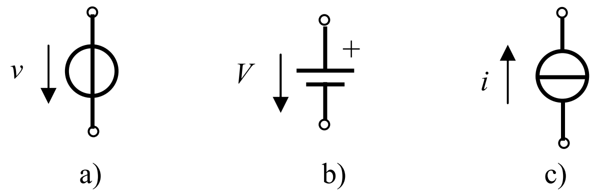


Fig. 1.1.2 Symbols for signal sources:
a) voltage; b) dc voltage; c) current.

The sign "+" is occasionally used next to one of the source's terminals to indicate the positive terminal. To differentiate through notations the different types of signals we will use the following convention (see also Fig. 1.1.3):

- only continuous signal (dc value) – uppercase variable and uppercase subscript V_S, I_S ;
- only time-varying signal – lowercase variable and lowercase subscript v_s, i_s;
- total instantaneous signal (continuous plus time-varying component) – lowercase variable and uppercase subscript v_S, i_S.

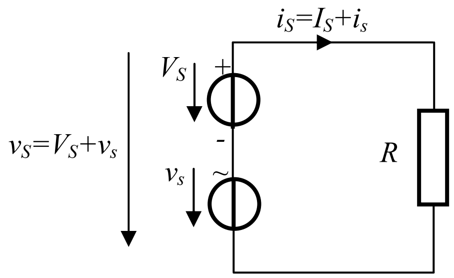


Fig. 1.1.3 Signal notation.

Considering V_S=5V and $v_s(t)=3sin\omega t$[V], the total signal is $v_S(t)=5+3sin\omega t$[V]. This voltage is also called a sinusoidal voltage having the amplitude of 3V and dc level of 5V. Its waveform is presented in Fig. 1.1.4.

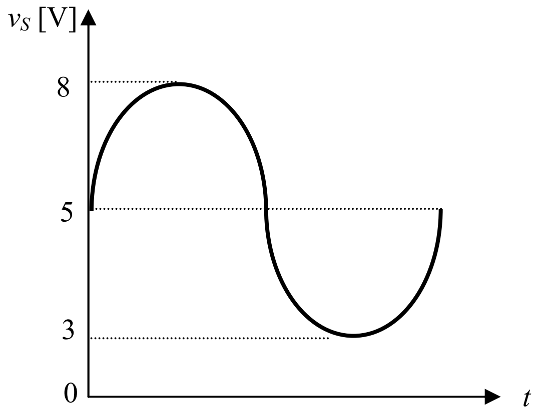


Fig.1.1.4 v_S=5+3$sin\omega t$ [V].

1.2 Relations and Theorems of Electric Circuits

1.2.1 Ohm's Law

Ohm's law, named after its discoverer, the German physicist Georg Simon Ohm, states the relation of proportionality between the voltage drop across a resistor and the current through that resistor, the resistance R being the factor of proportionality.

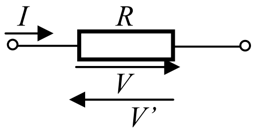


Fig. 1.2.1 Ohm's law illustration.

$$V=RI$$

If the arbitrary chosen directions for the voltage and current are opposite, than a minus sign appears in the Ohm's law relation (Fig. 1.2.1).

$$V'= - RI$$

1.2.2 Kirchhoff's Theorems

Gustav Robert Kirchhoff (a German physicist), enunciated two fundamental theorems for the analysis of the electric circuits.

- Kirchhoff's first theorem or *Kirchhoff's current law* (KCL):
  "The algebraic sum of all the currents entering and exiting any circuit node is zero at every instant."

In this sum the currents that enter the node and those who exit it will have opposite signs. For the circuit node presented in Fig. 1.2.2, the KCL is expressed as: $I_1+I_2-I_3=0$

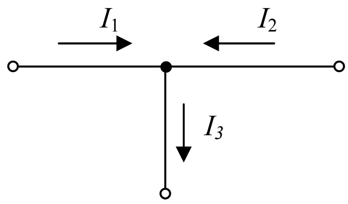


Fig. 1.2.2 Exemplification of KCL.

Note: KCL can also be interpreted as: in any circuit node there is neither power consumption nor power generation, all the current that enters a node has to exit it.

- Kirchhoff's second law or *Kirchhoff's voltage law* (**KVL**):
  "The algebraic sum of all the voltages around any closed circuit loop is zero at every instant."

For applying this theorem, one should choose an arbitrary direction of going over the circuit loop. The voltages that have the same direction as the direction of going over the loop will have the plus sign and the others the minus sign. For voltage sources we shall consider the voltage across its terminals and not the electromotive voltage. The expression of the KVL for the circuit in Fig. 1.2.3 is (the direction of going over the loop is clockwise):

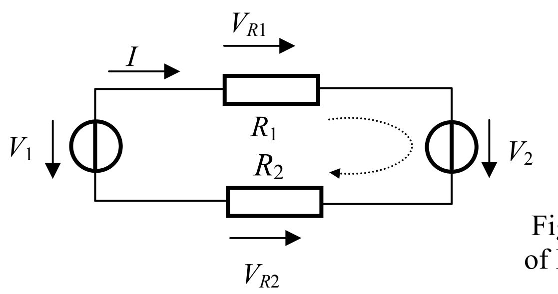


Fig. 1.2.3 Exemplification of KVL.

$$-V_1+V_{R1}+V_2-V_{R2}=0$$

Or, if we consider the current I through the circuit:

$$-V_1+R_1I+V_2+IR_2=0$$

1.2.3 Resistor Connections

- **The Series Connection**

Two or more resistors are series connected if the same current flows through all of them (Fig.1.2.4).

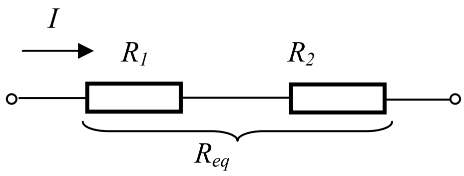


Fig. 1.2.4 Series connection of two resistors.

$$R_{eq}=R_1+R_2$$

By series connection of two or more resistors we obtain a larger equivalent resistance than any of the individual resistances.

- **The Parallel Connection**

Two or more resistors are parallel connected if the same voltage appears across each resistor, as shown in Fig. 1.2.5.

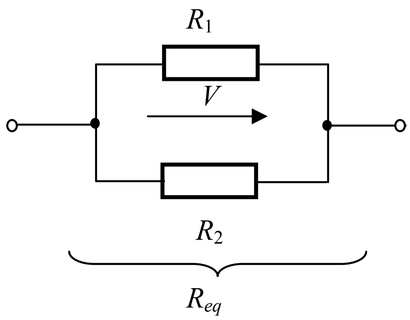


Fig.1.2.5 Parallel connection of two resistors.

$$R_{eq}=\frac{R_1R_2}{R_1+R_2}$$

The proof of this relation will be left for the amusement of the reader. By the parallel connection of two or more resistors we obtain a smaller equivalent resistance than any of the individual resistances.

For n resistors (R_i) connected in parallel, the equivalent resistance will be determined using the formula:

$$\frac{1}{R_{eq}}=\sum_{i=1}^{n}\frac{1}{R_i}$$

Tricks:

- The equivalent series (parallel) resistance of a high value resistor with a much smaller value resistor can be considered equal to the higher (smaller) resistance.
  For R_1=100KΩ and R_2=1KΩ, we have:
  series connection: $R_{eq}\approx$100kΩ; parallel connection: $R_{eq}\approx$1KΩ.
- For calculating the equivalent parallel resistance for a resistor of 10kΩ and one of 20kΩ we can consider the 10kΩ as being two resistances of 20kΩ in parallel. So we have three resistors of 20kΩ in parallel. The equivalent resistance is now very easy to calculate: 20kΩ/3=6.67kΩ.

These tricks are very useful because they ease the calculation so that we can focus on the analysis and design of the circuits. It is desirable to try to avoid the temptation of calculating the values of the resistances and other circuit elements to many significant places. There are at least two reasons for this:

a) the components themselves have a finite precision (typically the resistors have a 5% or 1% tolerance, the active devices parameters suffer from manufacturing dispersion, etc.);
b) a good circuit design is greatly insensible at the very precise values of the components (there are exceptions, as always).

We can obtain a better and quicker understanding of the circuits if we get into the habit of mentally making approximate computations instead of looking at precise numbers, with lots of decimals on the display of a calculator.

1.2.4 Resistive Dividers

- **The Voltage Divider**

The voltage divider is one of the most widespread electronic circuit fragments. Any real electronic circuit contains voltage dividers. It produces at the output a predictable fraction of the input voltage, as one can see in Fig. 1.2.6.

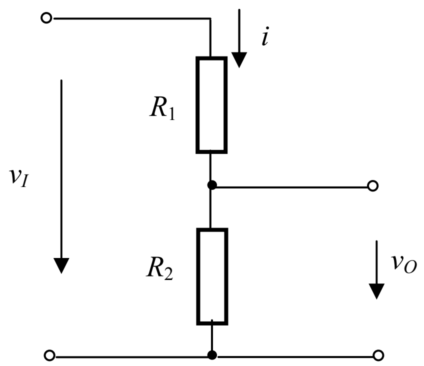


Fig. 1.2.6 The voltage divider.

$$i = \frac{v_I}{R_1 + R_2}$$

$$v_O = iR_2$$

$$v_O = \frac{R_2}{R_1 + R_2} v_I$$

The output voltage is proportional with the resistance across which we measure it and inversely proportional with the sum of the resistances. If the input voltage remains the same and the value of R_2 increases, the output voltage increases as well.

Trick: If one is in need for a voltage divider that has to supply to the output v_O=5V from a v_I=15V, we have only one equation and two unknowns R_1 and R_2. It would be a good idea (unless there are no other restrictions) to impose the current through the divider I=1mA, in this case the sum of the resistances in KΩ is numerically equal with the input voltage and the value of R_2 is numerically equal with the output voltage v_O. So it results R_2=5kΩ and R_1=15KΩ-5KΩ=10KΩ.

The simplest adjustable voltage divider can be build up using a single adjustable resistor, called potentiometer, which allows the adjustment of the dividing factor (the fraction of the input voltage that we obtain at the output) between 0 and 1 (Fig. 1.2.7).

The most common application of this divider is the volume control in an audio amplifier.

Connecting a fixed value resistor in series with the potentiometer can reduce the range of adjustability of the division ratio.

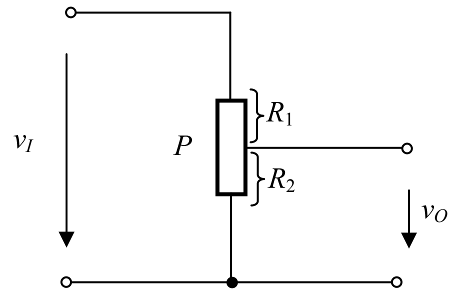


Fig.1.2.7 Adjustable voltage divider.

Example 1.2.1:

How does the circuit of a voltage divider, with the division ratio adjustable in the range [0.5; 1] look like? What are the values of the elements in the circuit?

Solution:

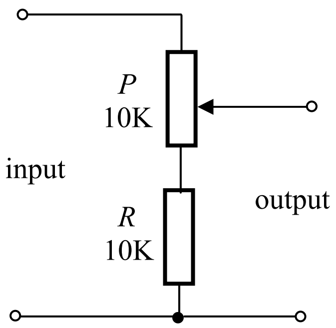


Fig. 1.2.8 Adjustable divider in the range [0.5;1].

- **The Current Divider**

The circuit of a current divider is presented in Fig. 1.2.9.

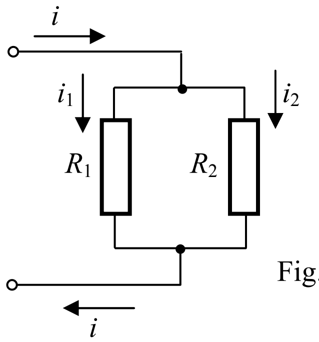


Fig. 1.2.9 The current divider.

KCL: $i=i_1+i_2$
KVL: $i_1R_1 - i_2R_2=0$

Solving the above system of two linear equations with two unknowns, it results:

$$i_1 = \frac{R_2}{R_1+R_2}i \qquad \text{and} \qquad i_2 = \frac{R_1}{R_1+R_2}i$$

If we consider i as the input current than any of the i_1 and i_2 currents can be considered as the output. The output current is not proportional with the resistance through which it flows but with the other resistance from the divider. If we keep i constant and R_2 increases, the current i_1 increases and the current i_2 decreases.

1.2.5 Superposition Method

Besides the Kirchhoff's theorems, which are valid for a certain circuit, linear or nonlinear, for linear circuits there also are some specific methods that simplify in a great deal the computation. One of these is the superposition method.

An electronic circuit is linear if the response of the circuit to a sum of input signals equals the sum of responses when each input signal is assumed to act alone in the circuit.

If $f(x_1)$ is the circuit's response applying the signal x_1, and $f(x_2)$ is the response applying the signal x_2, than the response of the circuit applying the sum of signals x_1+x_2 is $f(x_1)+f(x_2)$. The response of a linear circuit driven with a sinusoidal signal will always be sinusoidal, even if the amplitude and the phase are modified.

As an example, let us first consider the circuit in Fig. 1.2.10a), where the applied signals are the current sources i_1 and i_2, and the output signal is v_o.

According to Ohm's law: $v_o = Ri$; or we say that the circuit's function is:

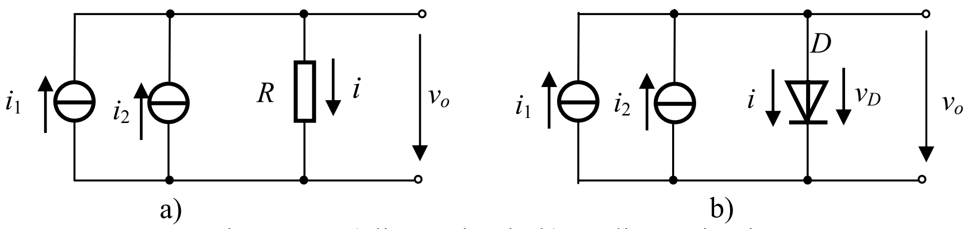


Fig. 1.2.10 a) linear circuit; b) nonlinear circuit.

$$f_L(x) = Rx$$

$$x_1 = i_1; \quad x_2 = i_2;$$

$$f_L(x_1) = Ri_1; \quad f_L(x_2) = Ri_2$$

$$f_L(x_1) + f_L(x_2) = Ri_1 + Ri_2$$

$$f_L(x_1 + x_2) = f_L(i_1 + i_2) = R(i_1 + i_2) = Ri_1 + Ri_2$$

We notice that:

$$f_L(x_1 + x_2) = f_L(x_1) + f_L(x_2)$$

so the circuit is linear.

For the circuit in Fig. 1.2.10 b) the current-voltage relation is of exponential type:

$$i = I_S e^{\frac{v_D}{V_T}}$$

where I_S is the saturation current and V_T is the thermal voltage.

$$v_O = v_D = V_T \ln \frac{i}{I_S}$$

so the circuit function is:

$$f_N(x) = V_T \ln \frac{x}{I_S}$$

$$f_N(i_1) = V_T \ln \frac{i_1}{I_S}; \quad f_N(i_2) = V_T \ln \frac{i_2}{I_S}$$

$$f_N(i_1 + i_2) = V_T \ln \frac{i_1 + i_2}{I_S}$$

It is obvious that:

$$V_T \ln \frac{i_1}{I_S} + V_T \ln \frac{i_2}{I_S} \neq V_T \ln \frac{i_1 + i_2}{I_S}$$

In other words:

$$f_N(i_1) + f_N(i_2) \neq f_N(i_1 + i_2)$$

Therefore, the circuit is a nonlinear one.

The superposition method consists of the following: To analyze an active linear circuit with more than one source, one separately compute the circuit response considering all the sources but one set to zero. The algebraic sum of all these partial responses is the complete response of the circuit.

Example 1.2.2

Let us determine the voltage v_O for the circuit given in Fig. 1.2.11a) using the superposition method.

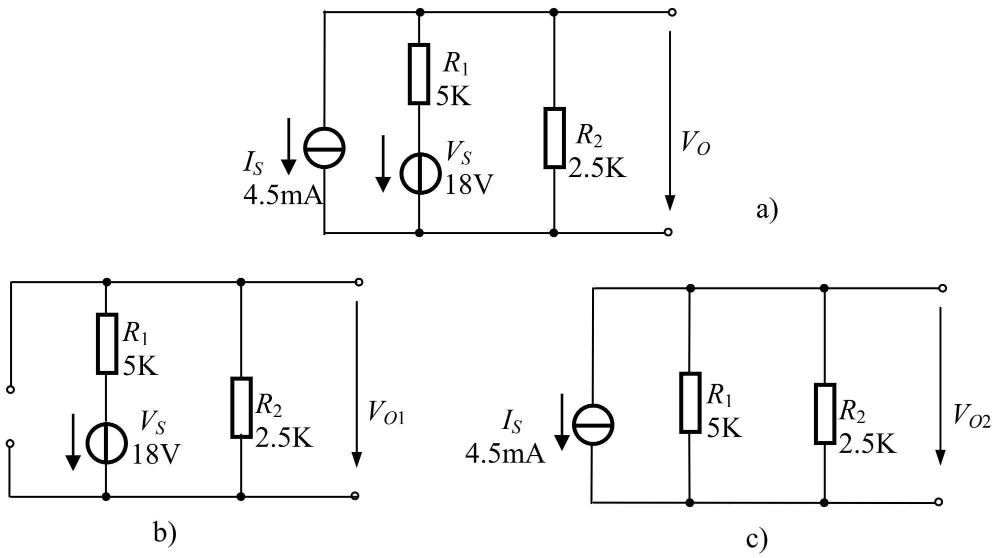


Fig.1.2.11 Application of the superposition method.

Due to the two sources, we will have two distinct situations:

1) Setting to zero the current source, it results the equivalent circuit in Fig. 1.2.11b)

$$V_{O1} = \frac{R_2}{R_1 + R_2} V_S = \frac{2.5}{5 + 2.5} \cdot 18 = 6\,\text{V}$$

2) Setting to zero the voltage source we obtain the equivalent circuit in Fig. 1.2.11c)

$$V_{O2} = -\frac{R_1 R_2}{R_1 + R_2} \cdot I_S = -\frac{5 \cdot 2.5}{5 + 2.5} \cdot 3 = -5\,\mathrm{V}$$

The total output voltage of the circuit is:

$$V_O = V_{O1} + V_{O2} = 6 + (-5) = 1\,\mathrm{V}$$

1.2.6 Thévenin's Theorem

Thevenin's theorem states that any network of resistors and sources can be replaced with an equivalent circuit of a single voltage source, v_{Th}, in series with a single resistor, R_{Th}. The theorem is also called *the equivalent voltage generator theorem.*

How do you figure out the values of the equivalent voltage source and equivalent resistor? Quite easy: v_{Th} is determined as being the open-circuit output voltage of the given circuit (without a load connected between its terminals); R_{Th} is the equivalent resistance of the given circuit with all the sources set to zero.

A source set to zero means: for a voltage source, the voltage across its terminals is zero (equivalent with a short-circuit) and for a current source, the current through the source is zero (equivalent with an open-circuit) as one can see in Fig. 1.2.12.

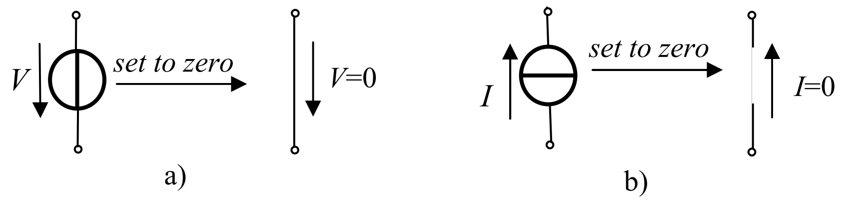


Fig. 1.2.12 Set to zero a: a) voltage source; b) current source.

Example 1.2.3

Applying Thévenin's theorem for the circuit in Fig. 1.2.13a) we obtain the equivalent circuit (a real voltage source) shown in Fig. 1.2.13b)[Mir83].

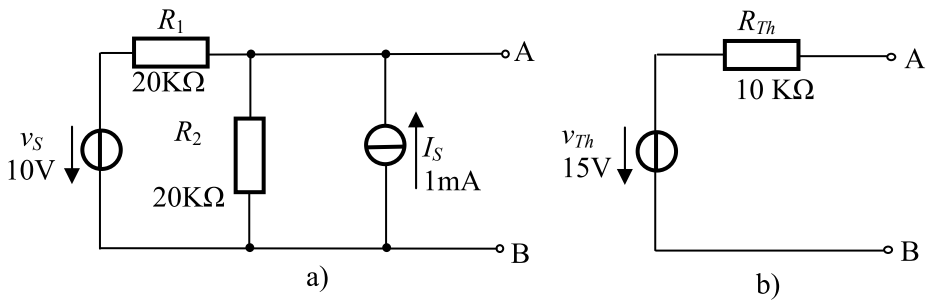


Fig. 1.2.13 Application of the Thévenin's theorem
a) the initial circuit; b) the equivalent circuit.

Applying the superposition theorem we have the voltage v_{Th} measured between the points A and B (Fig. 1.2.13a)):

$$v_{Th} = \frac{R_2}{R_1 + R_2} v_S + \frac{R_1 R_2}{R_1 + R_2} I_S = 5+10=15\text{V}$$

The equivalent circuit to deduce R_{Th} with the sources set to zero is the one in Fig. 1.2.14.

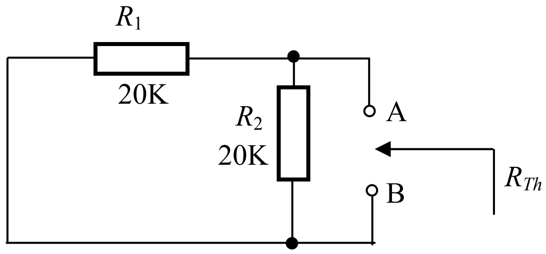


Fig. 1.2.14 The equivalent circuit for R_{Th}

$$R_{Th}=R_1 \parallel R_2=10\text{K}\Omega$$

R_{Th} also has another meaning, namely it represents the resistance seen between the two terminals of the circuit and it is called the output resistance (or internal resistance). This resistance can also be calculated by determining the short-circuit current at the AB terminals (Fig. 1.2.13a).

$$R_{Th} = \frac{v_{Th}}{i_{sc}}$$

It is worth mentioning that there also exists the equivalent current generator theorem (Norton's theorem), but because it is rarely used we do not present it in this book. If some of you are interested in, recommended readings are [Mir83] and [Şor82].

1.2.7 Millman's Theorem

Millman's theorem [Mir83] expresses the potential of a circuit node as a function of the conductances of the branches adjacent to that node and the potentials of the neighbor nodes. All these potentials are measured in respect with the same reference point (usually the ground).

For the circuit in Fig. 1.2.15, according to Millman's theorem we have the expression:

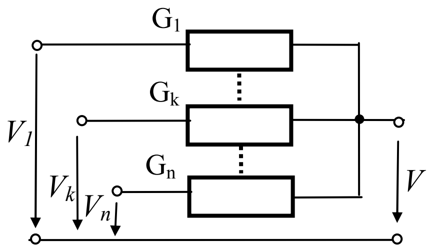


Fig.1.2.15 Illustration of Millman's theorem.

$$V = \frac{\sum_{k=1}^{n} V_k G_k}{\sum_{k=1}^{n} G_k}$$

We remind that $G = \frac{1}{R}$ is the conductance and its unit of measure is S (Siemens).

Example 1.2.4

Let us determine the voltage drop V across the resistor R for the circuit in Fig. 1.2.16.

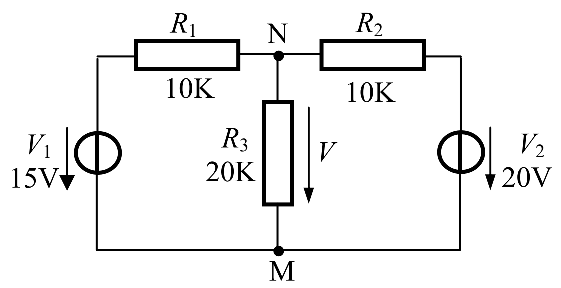


Fig. 1.2.16 Application of Millman's theorem.

We assume M as the reference point. The potential V_N in the point N is:

$$V_N = \frac{V_1 G_1 + V_2 G_2 + 0 \cdot G}{G_1 + G_2 + G}$$

$$V_N = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\frac{15}{10} + \frac{20}{20}}{\frac{1}{10} + \frac{1}{20} + \frac{1}{20}} = 12.5 \text{ V}$$

1.2.8 Power

The power (work done per unit of time) corresponding to a circuit is (Fig. 1.2.17):

$$P = VI$$

where V is the voltage across the circuit and I the current through the circuit. The unit of measure of the power is W (watt) with its submultiple 1mW=10^{-3}W and multiples 1KW=10^3W, 1MW=10^6W, 1GW=10^9W.

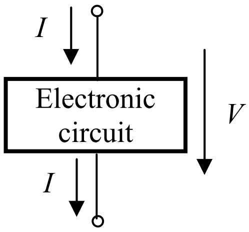


Fig. 1.2.17 The electrical power in a circuit

With the directions in the Fig. 1.2.17 (the voltage and the current have the same direction) we will consider:

$P>0$ – the power is consumed (dissipated);

$P<0$ – the power is generated.

Example 1.2.5

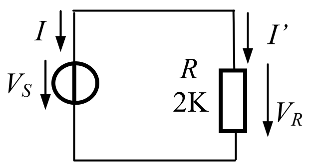


Fig. 1.2.18 Explanation for power computing.

$$I = -\frac{V_S}{R} = -5\,\text{mA}; \qquad I' = -I = 5\,\text{mA}$$

For the source is $P_S=V_SI$=10V·(-5mA)=-50mW; the power is generated by the source.

For the resistor is $P_R=V_RI'$=10V·5mA=50mW; the power is dissipated by the resistor.

In the above power calculation, on the source as well as on the resistor we've considered the same direction for the voltage and for the current (the receptors convention). If we consider different directions for the voltage and for the current (the generators convention), the interpretation of power changes, namely $P>0$ is the generated power and $P<0$ is the consumed power.

Please notice that *the power is conserving,* namely the generated power is equal, in modulus, with the consumed power.

For the calculation of the power dissipated by resistors, the following equivalent relations are obtained (using Ohm's law):

$$P = I^2 R \quad \text{and} \quad P = \frac{V^2}{R}$$

- **Power Transfer**

Here is an interesting problem: what load resistance R_L should one use to transfer maximum power in the load from a given source with its internal resistance R_S (Fig. 1.2.19)?

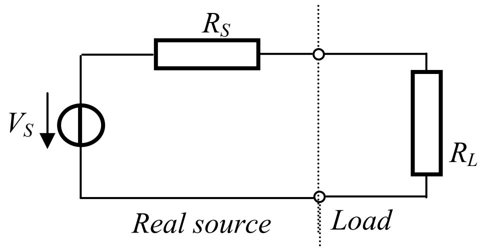


Fig.1.2.19 Maximum power transfer for R_L=R_S.

For extreme values of the load resistance:

$$R_L = 0 \qquad P_{RL} = I^2 R_L = 0$$

$$R_L = \infty \qquad P_{RL} = I^2 R_L = 0, \quad (I = 0)$$

There is an intermediary value for R for which the power transfer to the load is maximum and this value is:

$$R_L = R_S$$

Try to prove the above affirmation. Lest the previous affirmation should induce the wrong impression, we point out that, in practice, the electronic circuits are designed so that the load resistance is much greater than the internal source resistance.

We mention that the real source from the previous example can be the model of an equivalent Thévenin circuit output, for example of an amplifier.

1.3 The Capacitor and the Inductor

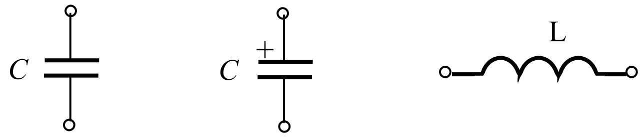
Fig. 1.3.1 presents the symbols for the capacitor and for the inductor:

The unit of measure for the capacitance is the Farad (F), with its submultiples:

1μF = 10^{-6} F, 1nF = 10^{-9} F, 1pF = 10^{-12}F.

The unit of measure for the inductance is H (Henry), with its submultiples:

1mH= 10^{-3} H, 1μH = 10^{-6}H.



a) b)

Fig. 1.3.1 The symbol for: a) capacitor b) inductor.

1.3.1 Current – Voltage Relation

- **Capacitor**

A capacitor of C farads with v volts across its terminals has q coulombs of stored charge on one plate and $-q$ on the other.

$$q=Cv$$

As defining relation between the voltage across and the current through the capacitor, we use the differential equation:

$$i = C\frac{dv}{dt}$$

The rate of change of the voltage determines the current through the capacitor.

As an example, if the voltage variation is of 1V/s across a capacitor with C=1F, the current through the capacitor is of 1A. In other words, if a capacitor of 1F is supplied with a 1A current, its voltage changes by 1V per second.

Remarks:

- The greater the capacitance, the greater the current through the capacitor.
- The rate of change of the voltage across a capacitor cannot be infinite because these would need an infinite current through the capacitor. The abrupt potential variation on a plate is also integrally transmitted on the other plate.

- **Inductor**

If you understand the behavior of the capacitor you will not have any problems with the inductor, the two being dual circuit elements.

The following elements are dual:

resistance R – conductance G
capacitance C – inductance L
voltage v – current i

We can use *the duality principle*: the equations of two dual circuits, expressed in dual values have the same form. [Mir 83].

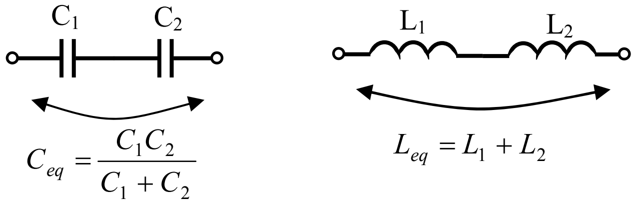
For an inductor, the rate of change of the current determines the voltage across. The current-voltage relation is:

$$v = L\frac{di}{dt}$$

If a voltage of 1V is applied across an inductor with the inductance of 1H, the current through the inductor changes by 1A per second.

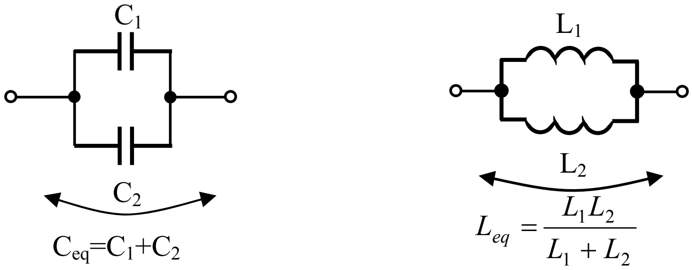
The inductor's behavior in a circuit can be deduced by duality with the capacitor's behavior.

1.3.2 Capacitor and Inductor Connections



a) b)

Fig.1.3.2 The series connection of: a) capacitors b) inductors.



a) b)

Fig. 1.3.3 The parallel connection of: a) capacitors b) inductors.

By the series (parallel) connection of the capacitors, we obtain a smaller (larger) capacitance.

1.3.3 The DC Behavior

1.3.3.1 RC Circuit with a Voltage Source

Let us consider a simple circuit with a single capacitor, a resistor and a dc voltage source connected in series, as shown in Fig. 1.3.4.

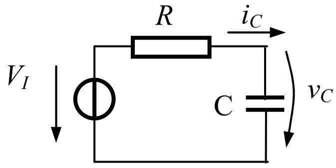


Fig. 1.3.4 RC series circuit with a dc voltage source.

We are interested in the time variation of the circuit quantities, particularly on the voltage across the capacitor $v_C(t)$ and on the current through the capacitor $i_C(t)$.

Application of the KVL gives:

$$V_I = R i_C + v_C$$

We also have the capacitor equation:

$$i_C = C\frac{dv_C}{dt}$$

$$V_I = RC\frac{dv_C}{dt} + v_C$$

We obtain an inhomogeneous, first order differential equation with the v_C as unknown:

$$RC\frac{dv_C}{dt} + v_C - V_I = 0$$

Solving the equation and using the boundary conditions:

$t=0$; $v_C=v_C(0)$ – the voltage across the capacitor at the initial moment;

$t=\infty$; $v_C=v_C(\infty)$ – the voltage across the capacitor at the final moment.

we obtain the solution:

$$v_C(t) = v_C(0)\cdot e^{-\frac{t}{\tau}} + (1-e^{-\frac{t}{\tau}})v_C(\infty)$$

where $\tau=RC$ is called the *time constant* of the circuit, having the second (s) as unit of measure.

For the circuit presented in Fig. 1.3.4 we consider the capacitor fully discharged at the initial moment, $v_C(0)=0$V. The final voltage that the capacitor reaches in an infinite long time ($t >> \tau$) is the voltage of the input source $v_C(\infty)=V_I$.

The equation that describes the time evolution of the voltage across the capacitor is:

$$v_C(t) = (1-e^{-\frac{t}{\tau}})V_I$$

The voltage waveform is presented in Fig. 1.3.5a):

At the moment $t=\tau$, the capacitor charges up to 63% of its final value. The capacitor can be considered fully charged after the time $t=5\tau$, when it reaches 99% of its final value.

The waveform of the current through the capacitor is presented in Fig. 1.3.5b).

$$i_C(t) = \frac{V_I - v_C(t)}{R}$$

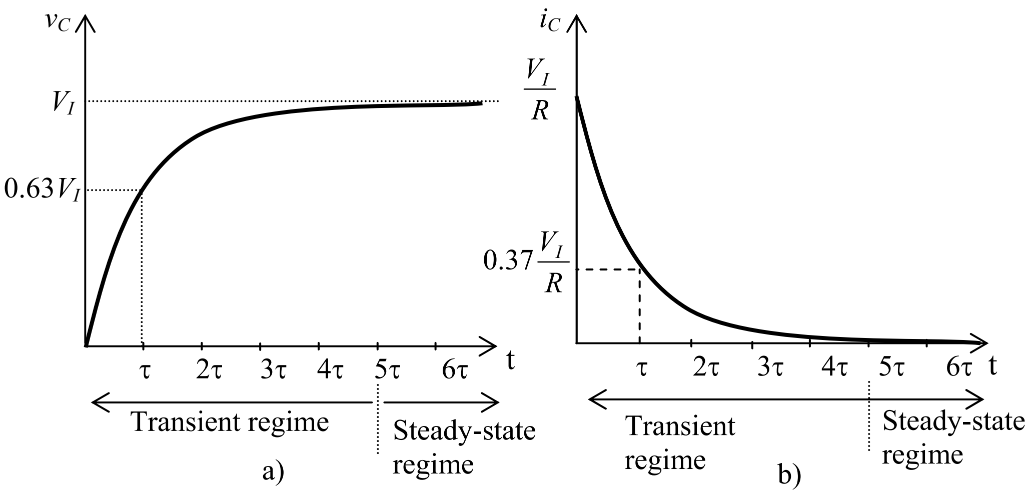


Fig.1.3.5 Voltage and current waveforms for the circuit in Fig. 1.3.4.

The current through the capacitor has its maximum value at the initial moment $i_C(0) = \frac{V_I}{R}$, because the entire source voltage falls across R. Once the voltage across the capacitor increases, the current through the circuit decreases, striving to zero after t=5τ.

In the RC circuit powered by a continuous voltage source, after the transient regime $t \in (0;\ 5\tau)$, follows the steady-state regime in which nothing happens, the current being zero.

Remark: The interpretation of the above affirmation can be: in dc, after the transient regime, the capacitor can be considered an open-circuit.

1.3.3.2 Charging the Capacitor at Constant Current

We consider a circuit containing a capacitor and a dc current source (Fig. 1.3.6)

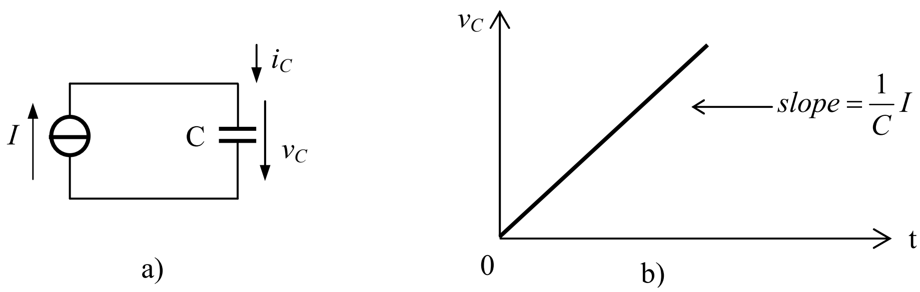


Fig. 1.3.6 A constant current source charging a capacitor: a) circuit; b) the voltage across the capacitor.

In the initial moment the capacitor is assumed to be discharged $v_C(0)$=0V. The current through the capacitor is constant in time $i_C(t)=I$, therefore we have:

$$v_C(t)=\frac{1}{C}\int_0^t i_C(t)dt$$

$$v_C(t)=\frac{1}{C}It$$

The voltage across the capacitor increases linearly in time (Fig. 1.3.6b)). If the circuit runs for a given duration, the voltage increases continuously running the risk of destruction of the capacitor or of the source. If at a certain moment, the direction of the current is reversed, the capacitor will discharge.

1.3.4 The AC Behavior

Circuits containing capacitors and inductors are more complicated than the pure resistive circuits, in that their behavior depends on frequency. For example, a voltage divider containing a capacitor or an inductor will have a frequency dependent division ratio.

The capacitor and the inductor are known as reactive circuit elements. For these elements, the *reactance X* is defined:

$$X_C=\frac{1}{\omega C}; \quad \text{for the capacitor}$$

$$X_L=\omega L; \quad \text{for the inductor}$$

The unit of measure for the reactance is the same as for the resistance $[\Omega]$.

Another term used with circuits containing reactive elements is that of *impedance,* denoted by *Z.* The impedance is a general term, it is a "generalized resistance" [Hor97]:

$$Z=R+j(X_L-X_C)$$

The reactance is the imaginary part of the impedance.

The capacitive impedance, Z_C and inductive impedance, Z_L are:

$$Z_C=R-jX_C; \qquad Z_L=R+jX_L;$$

where $j=\sqrt{-1}$.

For the ideal capacitor and inductor (R=0) the impedances are complex numbers:

$$Z_C=\frac{1}{j\omega C}=-j\frac{1}{\omega C}; \qquad Z_L=j\omega L$$

The reactance of a capacitor (inductor) decreases (increases) when the frequency increases. Therefore, in dc (f=0Hz) a capacitor is equivalent with a open-circuit ($X_C \to \infty$), while an inductor is equivalent with a short-circuit ($X_L \to 0$).

1.4 Generalization of Electric Circuits Theorems and Relations

When the analyzed circuits contain reactive elements (capacitors and inductors) all the relations and theorems of electric circuits (see paragraph 1.2), have to be reformulated. All these relations and theorems are still valid (respecting the applicability conditions) just that instead of the resistance we will use the impedance.

- **The Generalized Ohm's Law:**

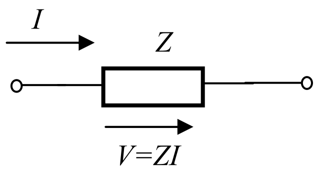


Fig.1.4.1 The generalization of Ohm's law.

If, for example, $Z=Z_c=\frac{1}{j\omega C}$ the voltage is given by $V=\frac{1}{2\pi f C}I$.

- **The connection of impedances** will replace the connection of resistors. For example two series impedances Z_1 and Z_2 have the equivalent impedance $Z=Z_1+Z_2$.

If:

$$Z_1=Z_C=\frac{1}{j\omega C} \quad \text{and} \quad Z_2=Z_L=j\omega L$$

$$Z=\frac{1}{j\omega C}+j\omega L=j(-\frac{1}{\omega C}+\omega L)$$

- **The dividers** can contain impedances instead of resistors. The calculus relationships are still valid, simply replacing R with Z. In general, the division ratio depends on frequency.
- **The superposition theorem** can be applied for circuits with C and L, using the corresponding impedances, but, don't forget, only for linear circuits.
- **Thevenin's theorem** for circuits where reactive elements are included must be restated: any network of resistors, capacitors, inductors, and sources can be replaced with an equivalent circuit of a single voltage source in series

with a single complex impedance.

- **Millman's theorem** expresses the potential of a circuit node as a function of the complex admittances of the branches adjacent to that node and the potentials of the neighbor nodes. All these potentials are measured towards the same reference point (usually the ground). By admittance we understand the inverse of the impedance, it is denoted by Y and its unit of measure is S (Siemens).

$$Y = \frac{1}{Z}$$

The admittance of a capacitor is $Y_C = j\omega C$ and of an inductor is $Y_L = \frac{1}{j\omega L}$.

1.5 Frequency Response

If a linear circuit is driven by a sine wave signal, the output will be a sinusoid of the same frequency. The output signal, however, could have a different amplitude and phase than the input signal. Thus, a circuit can be characterized in terms of the changes it causes in the amplitude and phase of sinusoids of various frequencies applied at the input.

1.5.1 The Complex Transfer Function

To evaluate the frequency response of a certain circuit one has to use the complex transfer function. The complex transfer function $F(j\omega)$ is a characteristic of the circuit and it completely describes its frequency response:

$$F(j\omega) = \frac{x_O(j\omega)}{x_I(j\omega)}$$

where $x_I(j\omega)$, $x_O(j\omega)$ denote the complex sinusoidal signals for the input and respectively for the output. These signals can be either voltages or currents.

The study of the frequency response of a circuit is in fact reduced at the study of the transfer function $F(j\omega)$.

Example 1.5.1

Derive the complex transfer function of the two-port network in Fig. 1.5.1.

Solution:

$$F(j\omega) = \frac{v_O(j\omega)}{v_I(j\omega)}$$

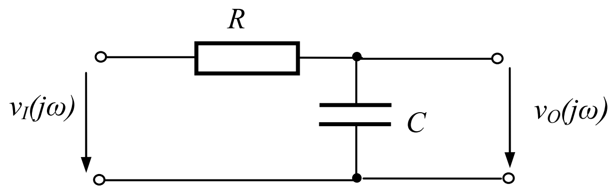


Fig. 1.5.1 *RC* circuit.

$v_O(j\omega)$ can be deduced considering a voltage divider driven by the input voltage $v_I(j\omega)$ that divides itself across the capacitor impedance Z_C and across the resistor's impedance Z_R.

$$v_O(j\omega) = \frac{Z_C}{Z_R + Z_C} v_I(j\omega)$$

$$F(j\omega) = \frac{Z_C}{Z_R + Z_C}$$

Substituting the impedances, we obtain:

$$F(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$F(j\omega) = \frac{1}{1 + j\omega RC}$$

The transfer function, being a complex number, is characterized by its magnitude and phase:

Magnitude: $|F(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$

Phase: $\Phi(\omega) = -arctg(\omega RC)$

Both the magnitude and the phase depends on the frequency (more precisely depends on angular frequency $\omega=2\pi f$). The graphical representation of these functions gives us a clear image on the frequency response of the circuit.

1.5.2 Frequency Response Representation

- **Logarithmic Scale**

For the representation of the magnitude of the transfer function, we use a system of coordinates having on the abscissa the frequency (or the angular frequency).

Taking in consideration the very wide range of the frequency, from hertz to megahertz, the use of a linear scale is almost impossible. That is why it is used a nonlinear, logarithmic scale, which has the property of “expanding” small values and “compressing” high values, allowing the representation of a very large variation interval.

The logarithmic scale is shown in Fig. 1.5.2.

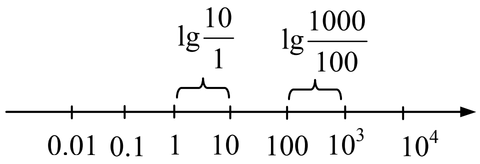


Fig.1.5.2 Logarithmic scale.

The origin of the axis is considered to be the unity value. On the axis, we write the values as powers of ten, but what we measure on the axis is the decimal logarithm of these values. The interval between two values, in a ratio of 10 (or 1/10), is called a decade. For example a decade is the interval between 1 and 10, or between 1000 and 100. On a logarithmic scale equal distances correspond to equal ratios.

- **Decibels**

On the transfer function magnitude axis one can use values expressed as a ratio, $|F(j\omega)|$ as well as values expressed in decibels, as a decimal logarithm of the ratio, $|F(j\omega)|_{dB}$:

$$|F(j\omega)|_{dB} = 20\lg|F(j\omega)|$$

The decibel is a sub multiple of the *bel*, which is the decimal logarithm of the ratio of two powers, when this ratio is equal with 10.

The correspondence between the transfer function magnitude values expressed as a ratio and expressed in decibels is shown in Fig. 1.5.3.

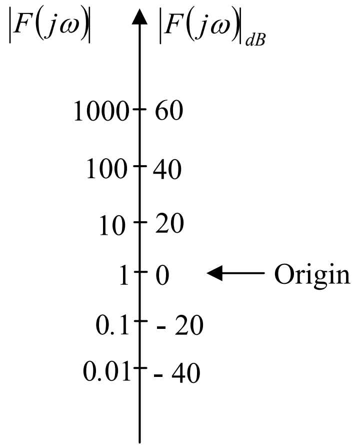


Fig.1.5.3 The correspondence between the transfer function magnitude values as a ratio and as decibels.

If the output signal is equal to the input one, $|F(j\omega)| = 1$, the magnitude of the transfer function in dB is $|F(j\omega)|_{dB} = 0$, which can be considered as the origin of the axis.

For an amplifier, when the output signal is greater than the input one, we have $|F(j\omega)|_{dB} > 0$, and for a circuit that attenuates, the output signal is smaller than the input one, and we have $|F(j\omega)|_{dB} < 0$.

- **Graphical representation**

The magnitude response for the circuit in Fig. 1.5.1 is given by:

$$|F(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

We will deal first with the asymptotic representation of this function, considering extreme values for ω:

$$\omega \to 0\ ; \qquad |F(j\omega)| = 1$$

$$\omega RC >> 1\ ; \qquad |F(j\omega)| \approx \frac{1}{\omega RC}$$

The magnitude curve is closely defined by the above two straight line asymptotes as it is represented in Fig. 1.5.4b) with broken line. The two asymptotes of the magnitude response curve meet at the *corner frequency*, or *break frequency* or *cutoff frequency* $\omega_0 = 2\pi f_0$, whose value is obtained by equaling the two functions.

$$1 = \frac{1}{\omega_0 RC} \qquad \omega_0 = \frac{1}{RC} \qquad f_0 = \frac{1}{2\pi\omega_0} = \frac{1}{2\pi RC}$$

Substituting $RC = \frac{1}{\omega_0}$, the transfer function can be written as:

$$F(j\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_o}}$$

with the magnitude: $|F(j\omega)| = \dfrac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_0}\right)^2}}$ and phase: $\Phi(\omega) = -arctg\dfrac{\omega}{\omega_0}$

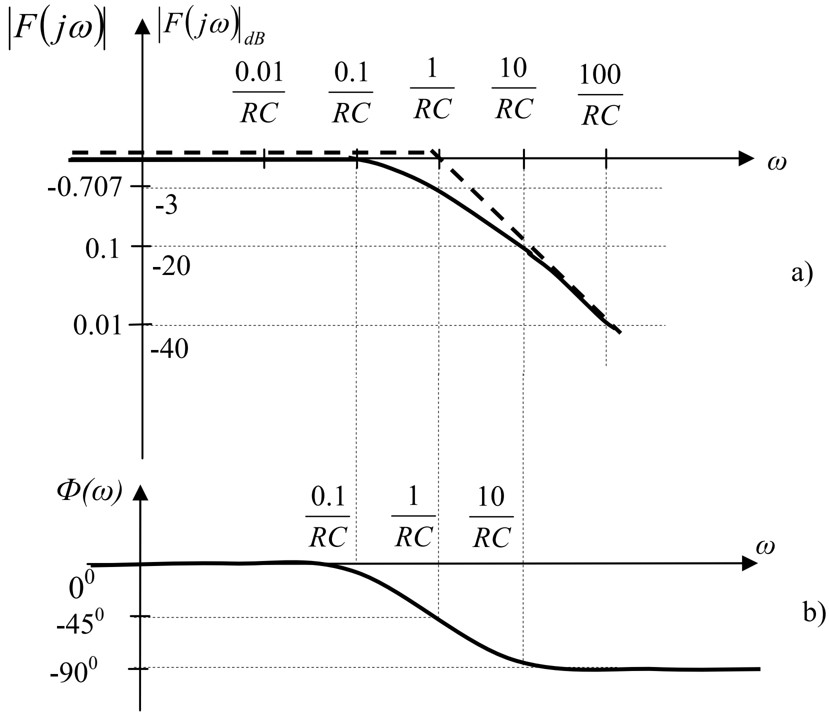


Fig.1.5.4 Frequency response of the LPF circuit in Fig.1.5.1:
a) the magnitude response; b) the phase response.

The real value of the magnitude at the break angular frequency is:

$$\left|F(j\omega_0)\right| = \frac{1}{\sqrt{1+\left(\frac{1}{RC}RC\right)^2}} = \frac{1}{\sqrt{2}} \cong 0.707$$

The real magnitude response curve is plotted with full line in Fig. 1.5.4a). If it is measured in decibels, the value of the magnitude at the corner frequency is:

$$\left|F(j\omega_0)\right|_{dB} = 20\lg\frac{1}{\sqrt{2}} = -3 \text{ dB}$$

For frequencies much smaller than the break frequency, the amplitude of the output signal is equal to the one of the input signal ($\left|F(j\omega)\right| = 1$). From the real magnitude response curve (Fig. 1.5.4a)) we notice that on approaching the break frequency the amplitude of the output signal decreases. It becomes, at the break frequency, 0.707 from the maximum signal amplitude. Therefore, when passing through the circuit, all the signals having the frequencies at most equal with the break frequency are attenuated with at most 30%, or with 3dB.

The band of frequency over which the magnitude of the transfer function is almost constant, to within a certain numbers of decibels (usually 3dB), is called the circuit ***bandwidth*** (or bandpass). Bandwidth of a signal is a measure of how rapidly it fluctuates with respect to time. Hence, the greater the bandwidth, the faster is the variation in the signal.

For our circuit the bandwidth is:

$$B = \frac{1}{2\pi RC}$$

Outside of the bandpass, the magnitude of the transfer function is inversely proportional with the angular frequency:

$$\left|F(j\omega)\right| = \frac{1}{\omega RC}$$

The slope of the magnitude response curve, outside the bandpass is - 20dB/decade. We say that the attenuation outside the bandpass is of - 20dB/decade.

The circuit that allows the low-frequency signals to pass and attenuates the high-frequency signals is called a *low pass filter,* LPF. There also are *high pass filters* HPF, band pass filters BPF and stop band filters SBF.

The phase of the transfer function is:

$$\Phi(\omega) = -arctg(\omega RC)$$

For: $\omega \to 0;\quad \Phi(\omega) = -arctg\, 0 = 0$

$\omega \to \infty;\quad \Phi(\omega) = -arctg\, \infty = -90^0$

$\omega = \omega_0;\quad \Phi(\omega) = -arctg\, 1 = -45^0$

The phase response of the transfer function is presented in the Fig. 1.5.4.b). The phase shift goes from 0^0 (at frequencies well below the break frequency) down to - 90^0 (at frequencies well above the break frequency), with a value of -45^0 at the break frequency.

A *RC* high pass filter is presented in Fig. 1.5.5, its frequency response being plotted in Fig. 1.5.6.

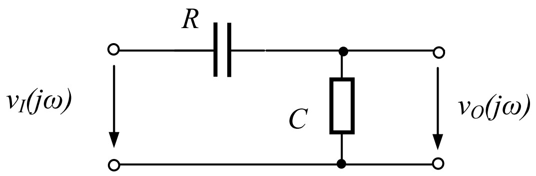


Fig.1.5.5 *RC* high pass filter.

The transfer function is:

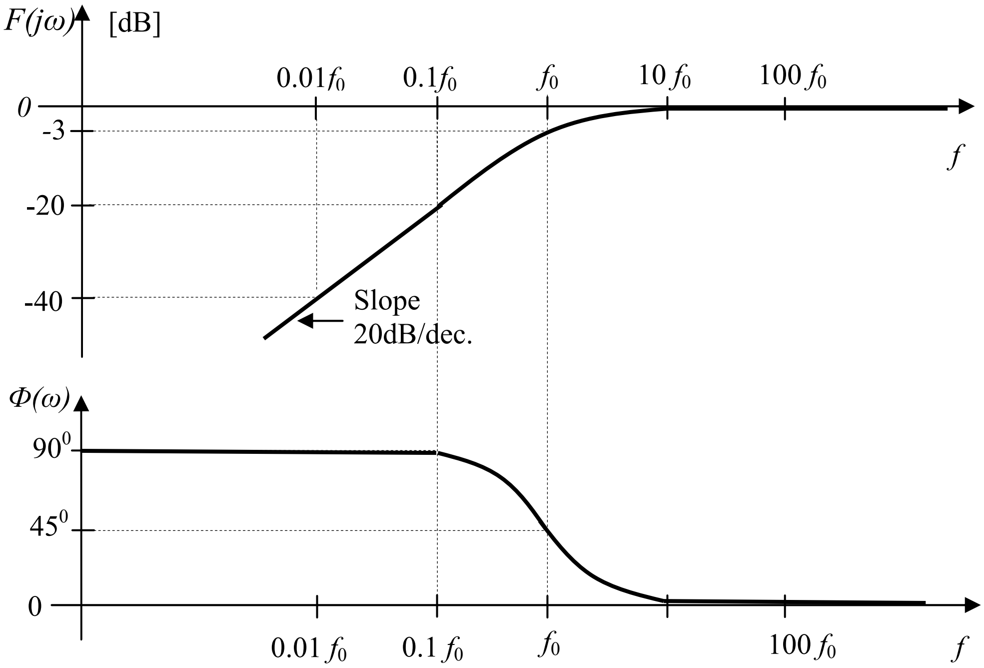


Fig. 1.5.6 Frequency response for a high pass filter.

$$F(j\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

$$|F(j\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}; \qquad \Phi(\omega) = 90 - arctg\omega RC$$

The break frequency is $f_0 = \dfrac{1}{2\pi RC}$, the bandpass at 3dB attenuation is: $B \in [f_0; \infty]$, the attenuation outside the band is 20dB/decade; the phase shift at the break frequency is 45^0.

Remark: For both LPF and HPF, at frequencies greater than $10f_0$ the slope of the magnitude response curve is with 20dB/decade smaller than the slope at frequencies smaller than $0.1f_0$. Also the phase shift is with 90^0 smaller. On the asymptotic representation of the magnitude response we say that the break frequency introduces an attenuation of 20dB/decade.

Example 1.5.2

Fig. 1.5.7 presents the magnitude response of a BPF amplifier. What are the bandpass and the gain in the bandpass of the amplifier? What is the expression of the transfer function?

Solution:

We notice that there are two break frequencies f_L=1KHz and f_H=10MHz where we have a gain of 37dB, with 3dB smaller than the maximum gain of 40dB. The

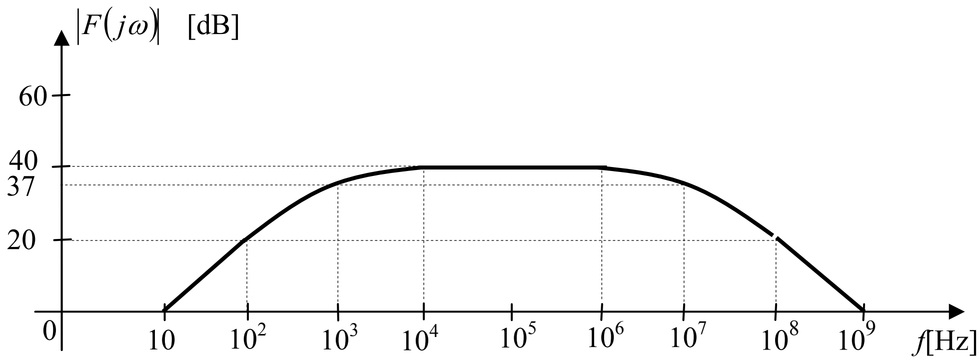


Fig. 1.5.7 Magnitude response of a band pass type amplifier.

bandpass is $B \in$[1KHz;10MHz]. The gain in the bandpass is 40dB, or measured as a ratio of the output and input signals it is 100.

The circuit transfer function has to present two break frequencies, so the denominator must be a product of the following type $\left(1+j\dfrac{\omega}{\omega_L}\right)\left(1+j\dfrac{\omega}{\omega_H}\right)$. The numerator has to have an expression of the form $j\dfrac{\omega}{\omega_1}$ which shows that we are coming from the origin ω=0, with the slope of 20dB/dec. ω_L and ω_H are read by inspection from the graphic:

$$\omega_L = 2\pi \cdot 10^3; \qquad \omega_H = 2\pi \cdot 10^7$$

ω_1 is determined using the condition that at the frequency f=10 Hz, $|F(j\omega)|_{dB} = 0$ dB

$$F(j\omega) = \frac{j\dfrac{\omega}{2\pi \cdot 10}}{\left(1+j\dfrac{\omega}{2\pi \cdot 10^3}\right)\left(1+j\dfrac{\omega}{2\pi \cdot 10^7}\right)} = \frac{j\dfrac{f}{10}}{\left(1+j\dfrac{f}{10^3}\right)\left(1+j\dfrac{f}{10^7}\right)}$$

We leave it to the fun of the reader to deduce the phase *Φ(ω)* and to sketch the phase response.